Game Master Information

World Builder's Cookbook

Compiled and annotated by John McMullen

This document contains equations, constants, and other goodies for world-building calculations. I've found explanations of the principles of world-building, but I often don't find the equations. So here they are.

Many of these are lifted from the non-copyrighted version of the sci.space FAQ. Others are taken from Stephen Gillett's excellent book, *World Building*, and the other sources named in the bibliography.

Equations

Acceleration and Distance

Where acceleration is constant, *d* is distance, *v* is velocity, and *t* is time.

Distance	$d = d_0 + vt + ½at^2$
Velocity	$v = v_0 + at$
Velocity squared	$v^2 = 2ad$
Acceleration on a cylinder of radius r and rotation period t:	$a = 4\pi^2 r/t^2$
Rotation period to give earth "gravity" on a cylinder of radius r	$t = 2\pi(r/9.8)^{½}$
Time to travel distance *d* at acceleration *a*, given constant acceleration half-way and constant deceleration half-way	$d = ½ at^2$ $t = 2 *(d/a)^{½}$

Basic Planetary Calculations

Surface gravity	$g = GM/r^2$
Surface gravity in earth units	$g = (d_P/d_E) \times (\rho_P/\rho_E)$
Escape velocity	$v_{esc} = 2^{½} \times v_c = (2GM/r)^{½}$
Orbital velocity	$v_{orbital} = (GM/a)^{½}$
Tides (in earth units) (Extremely variable based on undersea geography; this is only a general guideline)	$T = M/R^3$
Tides (in meters)	$T = (mR^4)/(MR^3)$
Orbital Energy of an object of mass m in an orbit around the sun (mass M) with semimajor axis a	$E = -G*M*m/(2a)$

Where

g	Acceleration due to gravity
G	Gravitational constant
M	Mass of body
d_P	Diameter of planet
d_E	Diameter of earth
ρ_P	Density of planet
ρ_E	Density of earth
a	Semimajor axis of orbit
R	Planetary radius

Stellar Information

Where:

Absolute magnitude	M
Apparent magnitude	m
Distance in parsecs	p
Luminosity (in solar units)	L
Intensity (solar constant = 1)	I
Distance of planet (in AU)	R
Diameter of star (Sol = 1)	D
Effective temperature (Sol)	T (5770 K)
Effective temperature (star)	t
Size in degrees	S

Absolute magnitude	$M = m+5 - 5\log p$
Apparent magnitude	$m = M + 5(\log_{10} p - 1)$
Luminosity	$L = 2.52^{(4.85 - M)}$
Apparent brightness	$I = L/R^2$
Stellar diameter	$D = L(T^2/t^2)$
Size in sky:	$S = 57.3D/R$

(for sizes ~ 20 degrees)

If you're using stars that are somewhat more extreme, you might want to calculate the bolometric magnitude instead. (Bolometric is the total amount of radiation put out by the star.) Add the correction values from this table to the magnitude of the star. For a more complete table showing additional classes, refer to Kaler 1997, p. 263.

Class	Main Sequence	Giants	Supergiants
O3	-4.3	-4.2	-4.0
B0	-3.00	-2.9	-2.7
A0	-0.15	-0.24	-0.3
F0	-0.01	0.01	0.14
G0	-0.10	-0.13	-0.1
K0	-0.24	-0.42	-0.38
M0	-1.21	-1.28	-1.3
M8	-4.0		

Table 1: Table of bolometric corrections for some stars. After Kaler 1997, p. 263.

If you really need to calculate it, there's an empirical formula and a calculator at http://www.go.ednet.ns.ca/~larry/astro/HR_diag.html.

Orbits

Period of a Circular Keplerian Orbit

This will hold true for small eccentricities.

$$T = 2\pi/(GM/a^3)^{1/2}$$

G	Gravitational constant
M	Mass of both bodies
r	Radius of orbit
a	Semimajor axis of orbit

A pair of planets *cannot* have stable orbits with periods whose ratios are simple fractions (2/1, 3/2, etc) unless they are *very* distant. If they do, they'll be pulling on each other in the *same* direction every time they get close to each other.

Orbital Velocities

Orbital velocities for orbits at a distance *r*:

a	Semimajor axis
μ	$G(m_1 + m_2)$
$v = [\mu/r]^{½}$	Circular orbit
$v = [\mu((2/r) - (1/a))]^{½}$	Elliptical orbit
$v = [\mu(2/r)]^{½}$	Parabolic orbit
$v = [\mu((2/r)+(1/a))]^{½}$	Hyperbolic orbit
$E = -Gm_1m_2/2a$	Energy of object in orbit

Eccentricities of orbits depending on orbit type, with semimajor axis a and semiminor axis b:

Circular orbit	$e = 0$
Elliptical orbit	$e < 1$
Parabolic	$e = 1$
Hyperbolic orbit	$e > 1$

Point of periapsis	$R_p = a(1-e)$
Point of apoapsis	$R_a = a(1+e)$
Note:	$2a = R_p + R_a$
Eccentricity of orbit	$e = R_p \times V_p 2 / GM$
Eccentricity of orbit	$e = (a^2 + b^2)^{½}/a$
Period of orbit	$P^2 = 4\pi^2/\mu a^3$
	$P = 2\pi/[\mu a^3]^{½}$

∆v Between Two Circular Orbits

Normally used for LEO to GEO calculations, this is T.N. Edelbaum's equation. Unless there are simplifications I'm not aware of, it should be valid for differences between any two circular orbits around the same primary:

$$\Delta V = (V_1^2 - 2V_1V_2 \cos(\pi/2\ \alpha) + V_2^2)^{1/2}$$

V_1	circular velocity initial orbit
V_2	circular velocity final orbit
α	plane change in degrees.

Orbit Limits

Roche's Limit

A satellite will break up if its orbit is within Roche's Limit:

$$L = 2.44\ r\ (density_p/density_s)^{1/3}$$

where

$density_p$	Density of planet
$density_s$	Density of satellite
r	Radius

Titius-Bode Law

GURPS Space uses a variant on this "law" (discovered by Titius, popularized by Bode) for placing planets.

Gillett says that current thinking is this is an example of tidal separations in the protocloud; it holds to lesser extents for moon systems as well, but with different parameters.

The classical formula, where r_n is the orbital distance for planet n:

$$r_n = (0.3 \times 2^n) + 0.4 \text{ AU}$$

A more general form, suitable for moons around planets, for planet *n*:

$$P_n = P_o A^n$$

Where:

P_o	Period of orbit of nth planet (traditionally in days)
P_o	Period of primary's rotation
A	semimajor axis of the orbit

Minimum Separation and Orbital Stability

There are a lot of factors that determine how closely two planets can orbit without throwing each other out, but a minimum separation is 3.5 times Hill's radius: (This section is particularly fussy and don't bother with it if you don't want to.) This material is my attempt at understanding some stuff that Brian Davis sent me; mistakes are mine, because I'm sure I don't understand it fully yet.

For these equations, the variables are:

a	planet's distance from the star (semimajor axis)

m planet s mass (or secondary body)
M star's mass (or primary body)

To calculate Hill's radius for a particular star/planet pair:

$$a\_hill = a\ (m / 3M)^{1/3}$$

Separation between two bodies should be *at least* three and a half times the larger of the feed limit or the chaos band.

The feed limit is the same as the Roche limit, 2.4 times the Hill radius. Basically, a planet will "crush and eat" anything orbiting within this radius:

$$Separation\_feed > 2.4\ a\ (m / M)^{(1/3)}$$

For smaller planets the chaotic perturbation band is larger than this limit:

$$Separation\_chaos > 1.5\ a\ (m / M)^{(2/7)}$$

Rotation and Tidal Locking

Rotation

Current thinking is that the rotation period varies tremendously; you can set whatever period you wish. At one extreme is about ninety minutes for earth-like planets; at the other extreme, a satellite may be *tide-locked* with its primary, always showing the same face (our moon is tide-locked to earth). The minimum time will depend on the density of the body (you don't want it to fly apart).

If the satellite orbits in a prograde motion (such as the moon), tidal friction will eventually slow the rotation of the planet and the satellite will move farther away. If the satellite orbits in a retrograde motion, tidal friction will speed up the rotation of the planet and the satellite will move closer in.

Tidal Locking

Very roughly speaking, planets inside the tidal locking limit will have one face locked towards the star. The tidal lock limit is in AU:

$$Tidal\ Lock\ limit = 0.0483\ (T\ M^2/\rho)^{1/6}$$

where

T Age of system in years
M Mass of star in solar masses
ρ Density of planet in kg/m^3

Planetary Insolation

Insolation of a planet determines approximately how much light it gets, and (in solar units) depends on the luminosity of the star and its distance. Brian Davis comments that recent work suggests, conservatively, that I must be between 0.53 and 1.1; see the "fudged temperature" for a more recent measurement.

Insolation (relative)	$I = L/D^2$
Luminosity of star	L
Distance from star	D

Luminosity is normally done in terms of solar luminosities, so the D is in AUs. See "Stellar Information" for more about luminosity and magnitude.

Intensity	$I = \sigma T^4$

Planetary Tides

This equation is reflects the forces between two bodies. The last theory I saw stated that tide heights in specific areas might be the result of standing waves formed by the shape of the ocean bottom—in other words, highly variable and individual. Still, this equation reflects the average magnitude.

$$Tide\ height\ (m) = (mR^4)/(MR^3)$$

m remote body's mass in kg
M central body's mass in kg
R distance between bodies in m

Barycentre Calculation

The barycentre is the center of mass between two bodies.

$$s = ((m * R) / (M + m))$$

Where:

s distance of barycentre from central mass [m]
m satellite mass [kg]
M central mass [kg]
R radius between the two bodies (from their centres) [m]

Rocket Equations

Classical rocket equation

Where d_v is the change in velocity, I_{sp} is the specific impulse of the engine, v_e is the exhaust velocity, x is the reaction mass, m_1 is the rocket mass excluding reaction mass, g is acceleration due to gravity on earth:

Exhaust velocity	$v_e = g\ I_{sp}$
Change in velocity	$\Delta V = v_e \times \ln((m_1+x)/m_1)$
Or: Ratio of masses	$(m_1+x)/m_1) = e^{(d/v)}$

Note that $(m_1+x)/m_1)$ is the ratio of the initial mass to the final mass.

The exponent d/v is change in velocity over exhaust velocity.

For a staged rocket where each stage has the same ratio R of initial to final mass and with *n* stages, the final delta-vee is:

$$Final\ \Delta V = n\ [v_e \ln(R)]$$

You may notice that's the same as the single stage orbit multiplied by *n*. Essentially, two stages give you twice the final velocity of a single stage rocket with the same mass ratio, and so on.

Relativistic equation

For constant acceleration:

Time (unaccel.)	$t_u = (c/a) \times \sinh(at/c)$
Distance	$d = (c^2/a) \times (\cosh(at/c)-1)$
Velocity	$v = c \times \tanh(at/c)$

Temperatures

Temperature of a blackbody:

Albedo A
Incident light (sun=1) I
Temp in degrees Kelvin T

$$T = 374(1-A)I^{1/4}$$

To allow for greenhouse gases, Gillett suggests a fudge factor of about 1.1 for habitable planets:

$$T = 374 \times 1.1(1-A)I^{1/4}$$

Intensity of blackbody per unit area:

Stefan-Boltzmann constant σ
Temperature, degrees K T

Hohmann Transfer Orbits

A Hohman transfer orbit is the minimum energy orbit to get from planet A to planet B, assuming they have circular Keplerian orbits. The orbit is circular, with a tangent at the perihelion of one planet and another tangent at the aphelion of the other.

Semimajor axis of planet 1 R_1
Semimajor axis of planet 2 R_2
Semimajor axis of the transfer orbit $a=(R_1+R_2)/2$

Once you have the semimajor axis, you know transfer time: it's *half* the orbital period for a circular Keplerian orbit of that radius (use equation above).

To calculate required ΔV, you need to know the orbital velocity for your transfer orbit at the points where it's tangential to the orbits of the departure and destination planets:

$$V = (2GM \times [1/r - 1/2a])^{1/2}$$

The transit time for a Hohmann transfer orbit is half of the orbit, or:

$$\frac{1}{4} \times P_1 \times (1+R_2/R_1)^{3/2}$$

Ignoring for now the problems of calculating the angle that the destination planet needs to subtend and calculating the launch date; sample calculations for Earth to Mars can be found at:

http://www.marsacademy.com/text/angplan.htm
http://www.marsacademy.com/text/ladate.htm

Constant Acceleration Transit

There's a second kind of easily-calculated, efficient orbit, one that assumes a constant low acceleration (the sort you'd expect from an ion drive or a solar sail).

The acceleration must be very much lower than R/P^2, where R is the distance from the sun and P is the period of the *outermost* planet. (Note however, that this is *extremely* low; the value of R/P^2 for Earth is 0.015 m/s^2; for Mars, it is 0.0065 m/s^2, or less than 7 ten-thousandths of a G.)

An acceptable approximation of the travel time is:

$$2\pi R_1/(aP_1) \times (1 - R_1/R_2)^{1/2}$$

Where R_1 and P_1 are the distance from the Sun and the period of the inner planet, R_2 the distance between the Sun and the outer planet and a the acceleration of the spacecraft. (Take care to use consistent units: If a is in m/s^2, P_1 must be in *seconds*.)

You can get a value good enough for story or RPG purposes by doubling $t=(2d/a)^{1/2}$, where *d* is half the distance to the other planet. For example, say that Mars to Earth is (2.279E11 – 1.496E11 meters) 7.83E10 meters, the closest approach. You have a solar sail that gives you 0.001 G acceleration, or 0.01 m/s^2. The time to accelerate half-way there is:

$$(7.83E10/0.01)^{1/2} = (7.83E12)^{1/2} = 2.8E6 \text{ seconds}$$

A little over 32 days. Assume the same time to decelerate, for a total Earth-to-Mars time of about 65 days.

A note from the website http://dutlsisa.lr.tudelft.nl/Propulsion/Data/V_increment_requirements.htm says:

"Transfer or trip time for constant thrust spiral is is calculated by dividing total propellant mass by mass flow. Total propellant mass is calculated using the rocket equation also known as Tsiolkowsky's equation. In case of negligible propellant mass (constant acceleration), transfer time can be calculated by dividing the velocity change by the acceleration."

Schwartzchild Radius

For a black hole of mass M, the Schwartzchild radius r is:

$$r = 2GM/c^2$$

Constants and Values

Some useful constants. Since it's sometimes easier to work things out in solar or terran equivalents, some physical data for our solar system is also included.

For game or story purposes, one or two significant digits is usually all you need, but I've gone to four here.

Constants

G (gravitational constant)	6.673E-11 Nm^2/kg^2
c (speed of light in vacuum)	2.998E8 m/s
Luminosity of sun	3.827E26 W
Solar constant (intensity@1 AU)	1370 W/m^2
Planck's constant *h*	6.6262E-34 J-s
"h bar" $h/(2\pi)$	1.055E34 J-s
Boltzmann's Constant *k*	1.381E-23 J/K
Stephann-Boltzman Constant σ	5.670E-8 $W/m^2/K$
Earth gravity acceleration	9.80665 m/s^2

Distances

One light year (meters)	9.461E15 m
	2.063E5 AU
One parsec (light years)	3.262 ly
	3.086E16 m
Mean earth-moon distance	3.844E8 m
Mean earth-sun distance (1 AU)	1.496E11 m
Mean radius of earth	1.371E6 m
Equatorial radius of earth	6.378E6 m
Mean Mercury-sun distance	5.79E10 m
	0.387 AU
Mean Venus-sun distance	1.082E11 m
	0.723 AU
Mean Mars-sun distance	2.279E11 m
	1.524 AU
Mean Jupiter-sun distance	7.783E11 m
	5.203 AU
Mean Saturn-sun distance	1.427E12 m
	9.539 AU
Mean Uranus-sun distance	1.8696E12 m
	19.182 AU
Mean Neptune-sun distance	4.4966E12 m
	30.058 AU
Mean Pluto-sun distance	5.9001E12 m
	39.44 AU

Masses and Densities

Mass of Sun	1.989E30 kg
Mass of Earth	5.974E24 kg
Mass of Moon	7.348E22 kg
Average density of Earth	5.5 g/cm^3
	5500 kg/m^3

Game Mechanics

The following information is relevant to a particular set of game mechanics, *CORPS* by BTRC.

Solar Sail info from VDS

The BTRC *Vehicle Design System* (VDS) has the power from a solar sail constant per square kilometer of sail (10 w at Earth orbit). Fiddling with the acceleration equation gives us these two versions:

$$a^2 = 2P/(5M)$$
$$M=2P/(5a^2)$$

Where *a* is the acceleration in meters/s^2, M is the mass of the vehicle in kilograms, and P is the power in watts. If you calculate the acceleration for a given vehicle at earth orbit (10 watts), the acceleration at other orbits is proportional to the distance in AU. (Calculating for another star is a different matter.)

Sails for different TLs have the following "characteristic accelerations" (acceleration with no payload):

TL	Power	Mass/km^2	Char accel	Load@0.01g
11	10w	413	0.089	39,587
12	10w	347	0.107	39,653
13	10w	296	0.116	39,704
14	10w	255	0.125	39,745
15	10w	222	0.134	39,777

Note that since the power is constant, the total mass moved per kilometer of sail is constant: 40,000 kilograms. The difference is simply how much of it is devoted to payload. Given a desired acceleration of 0.01g, a square kilometer of sail carries approximately 39,500 kilograms of payload.

Since the weight of the powertrain at TL 14 is 30/14 kilograms per kilowatt, and a square kilometer of sail produces only 1/100th of a kilowatt, the powertrain is ignored here, or is absorbed in the excess "payload" beyond 39,500 kilograms.

References

Equations and data were taken from the following references:

World-Building, Stephen L. Gillett, Writer's Digest Books, 1996.

Vehicle Design System, Greg Porter, Blacksburg Tactical Research Center, 1997.

"Making Believable Planets," Peter Jekel, *Strange Horizons* (http://www.strangehorizons.com/2002/20020225/planets.shtml)

Some posts in rec.arts.sf.science by Brian Davis (bdavis@pdnt.com) in a thread in December of 2000.

The constant acceleration formula came from MA Lloyd in a post to a *GURPS* mailing list archived at http://www.rollanet.org/~bennett/gmsf/relspc4.txt.

Bolometric Magnitude from Johnson, H.L.; Morgan, W.W. (1953): Astrophysical Journal, 117: 313.

Bolometric Magnitude reference from Kaler, James B. (1997): Stars and Their Spectra. Cambridge. (Corrected paperback ed.) 300 pp.

Hill radius data from a document by Brian Davis, emailed to me.

Still Under Construction