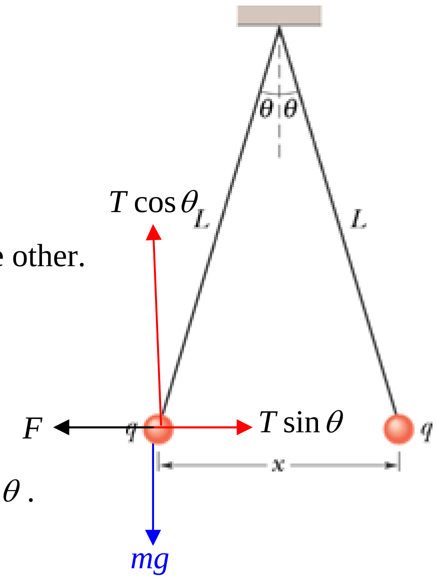
54. a) Let's prove this equation.

As we can see from this diagram,



$$T\sin\theta = F$$
$$T\cos\theta = mg$$

where F is the electrostatic force on one of the balls due to the other.

Dividing these two equations we get
$$\tan\theta = \frac{F}{mg}.$$

The question instructs us to use the approximation $\tan\theta \approx \sin\theta$.
$$\sin\theta = \frac{F}{mg}.$$

From the triangle, $\sin\theta = \dfrac{\frac{x}{2}}{L}$. The electrostatic force $F = k\dfrac{q^2}{x^2}$. Putting these two equations into the equation we have above, and organizing the variables leads to the result desired:
$$x = \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3} = \left(\frac{2kq^2 L}{mg}\right)^{1/3}.$$

b) $|q| = \sqrt{\dfrac{x^3 mg}{2kL}} = \sqrt{\dfrac{(0.05)^3(0.010)(9.8)}{2(8.99\times10^9)(1.20)}} = 2.4\times10^{-8}\,C$.

55. a) The neutral ball will be attracted by the charged ball. The two balls touch each other, and as a result, the net charge q gets shared between the two balls. Now each ball has a charge of $q/2$.

b) The new equilibrium separation is
$$x_{new} = \left(\frac{2k\left(\frac{q}{2}\right)^2 L}{mg}\right)^{1/3} = \left(\frac{2k\left(\frac{1}{4}q^2\right)L}{mg}\right)^{1/3} = \left(\frac{2k\left(\frac{1}{4}\times\frac{x^3 mg}{2kL}\right)L}{mg}\right)^{1/3} = \frac{x}{\sqrt[3]{4}} = \frac{5cm}{\sqrt[3]{4}} = 3.1cm$$