

25. Electric field lines go from + to -. If we sketch an electric field line in each quadrant that are symmetrical about the x and y axes, we see that the x-components of the electric field lines cancel out and that we are left with lines pointing in the negative y-direction. In the figure below, the components of the electric fields in each quadrant are shown with colored arrows.



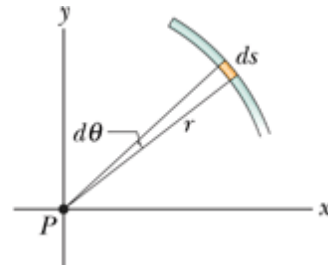
Let's consider only one quadrant when making the calculations. Then, the magnitude of the resulting electric field due to the full circle is four times the magnitude of the y-component of the electric field due to one quadrant of the circle.

The line charge density, $\lambda = \frac{+q}{2\pi r/2} = \frac{q}{\pi r}$ (considering only the upper half of the circle).

Then, the charge of an arc of length ds is $dq = \lambda ds$.

If θ is taken in radians, then $ds = r d\theta$. Therefore, $dq = \lambda r d\theta$. The expression dE is thus

$$dE = k \frac{dq}{r^2} = k \frac{\lambda r d\theta}{r^2} = \frac{k\lambda}{r} d\theta.$$



As we are looking for only the y-component, the expression becomes $\frac{k\lambda}{r} \sin \theta d\theta$.

Integrating this expression, we get

$$\int_0^{\pi/2} \frac{k\lambda}{r} \sin \theta d\theta = -\frac{k\lambda}{r} \cos \theta \Big|_0^{\pi/2} = \frac{k\lambda}{r},$$

which is the magnitude of the y-component of electric field due to only one quadrant

(one-fourth) of the circle. Multiplying this result by 4, and substituting in $\lambda = \frac{q}{\pi r}$, we get

$$E_{y(\text{total})} = \frac{4kq}{\pi r^2} = \frac{4(8.99 \times 10^9)(15.0 \times 10^{-12})}{\pi(0.0850^2)} = 23.8 \frac{N}{C}.$$