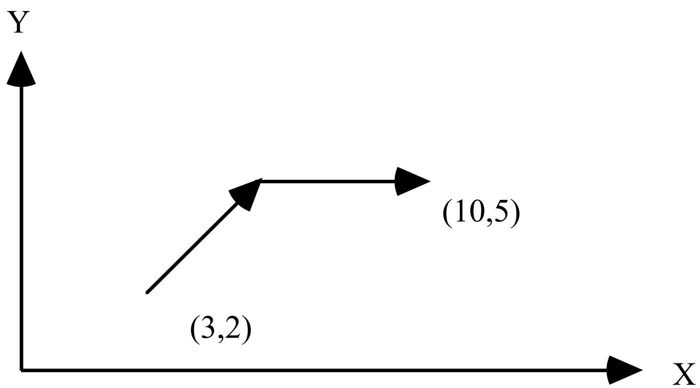
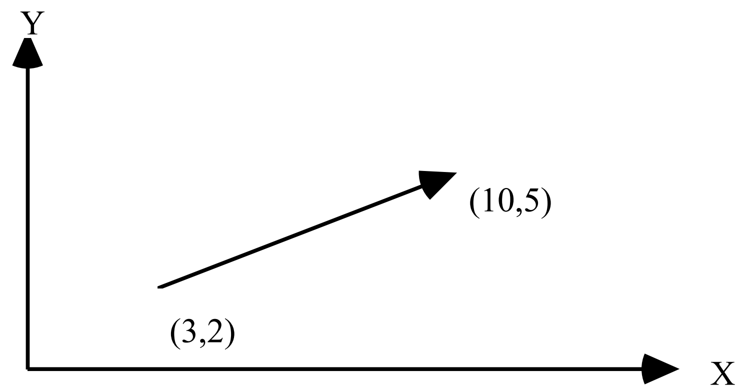
# INTERPOLATION

Control multiple axes simultaneously to move on a line, a circle, or a curve.




**Point-to-point control path**




**Linear path**

$$V_x = 6\frac{(10-3)}{\sqrt{(10-3)^2 + (5-2)^2}} = 6\frac{7}{\sqrt{49+9}} = 5.5149$$

$$V_y = 6\frac{(5-2)}{\sqrt{(10-3)^2 + (5-2)^2}} = 6\frac{3}{\sqrt{49+9}} = 2.3635$$

# INTERPOLATORS

Most common interpolators are: linear and circular

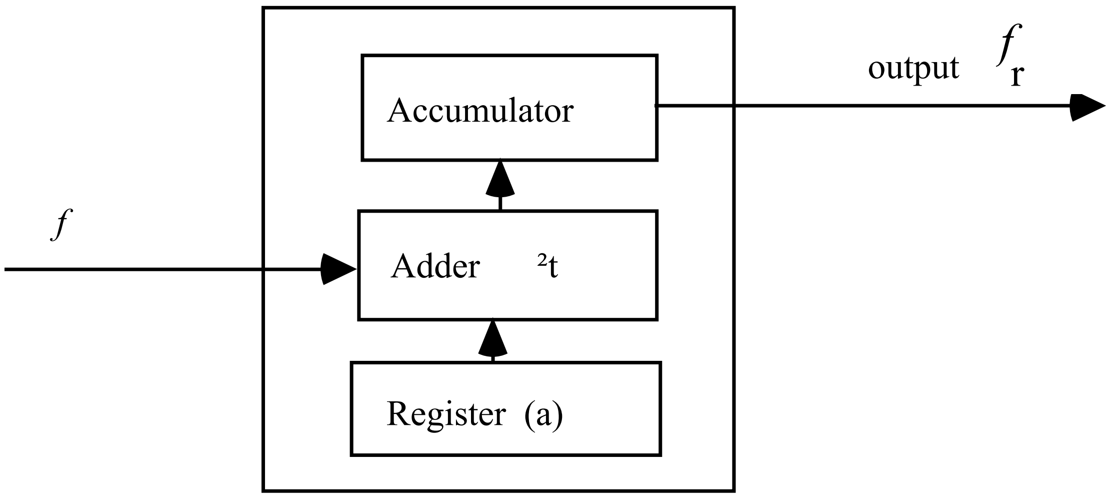
Since interpolation is right above the servo level, speed is critical, and the process must not involve excessive computation.

Traditional NC interpolators: Digital Differential Analyzer (DDA)

Higher order curves, such as Bezier's curve, use off-line approximation algorithms to break the curves into linear or circular segments.

# A DDA

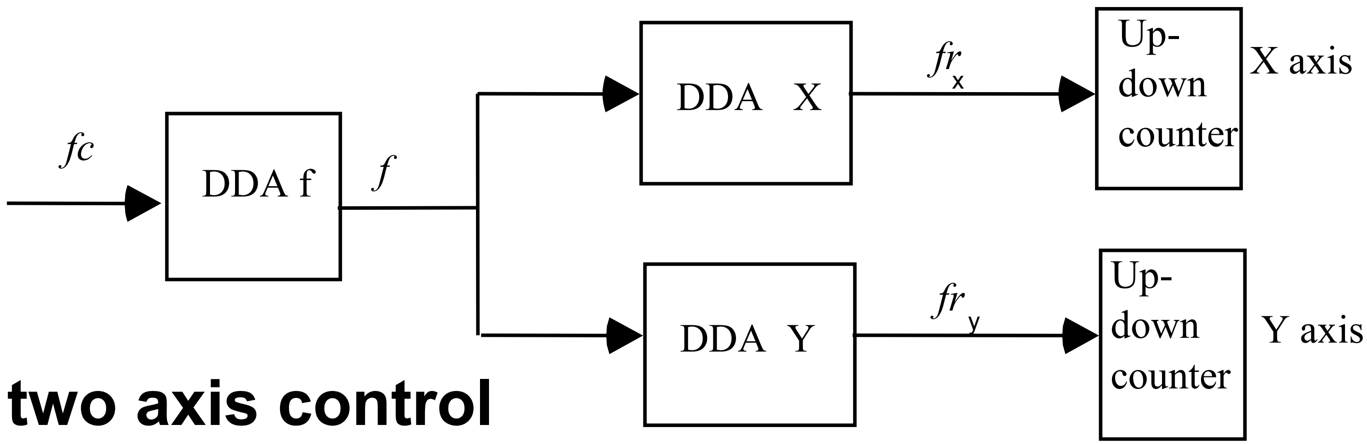
**Each time a pulse is received, the value of the register (a value) is added to the accumulator. The overflow bit of the accumulator is output to the motor control.**




$$f_r = \frac{a\,f}{2^N}$$

**N: accumulator width, bit**

# LINEAR INTERPOLATOR




## A two axis control

$$f = \frac{a_f f_c}{2^{N_f}}$$

**Feedrate control**

$$f_r = \frac{a_f f_c}{2^{N_f}} \frac{a}{2^N} = \frac{a a_f}{2^{(N_f + N)}} f_c$$

**Output to axis control**

# LINEAR INTERPOLATOR (continue)

**Since feedrate is the linear speed, how to convert it in *Vx* and *Vy* without using a computer?**

$$f_{rx} = V_f \frac{\Delta x}{\sqrt{\Delta x^2 + \Delta y^2}}$$

$$\frac{a_x a_f}{2^{(N_f + N)}} f_c = V_f \frac{\Delta x}{\sqrt{\Delta x^2 + \Delta y^2}}$$

Set $ax$ to $\Delta x$ ($ay = \Delta y$)

$$\frac{a_f}{2^{(N_f + N)}} f_c = \frac{V_f}{\sqrt{\Delta x^2 + \Delta y^2}}$$

$$a_f = \frac{V_f}{\sqrt{\Delta x^2 + \Delta y^2}} \frac{2^{(N_f + N)}}{f_c}$$

$\frac{2^{(N_f + N)}}{f_c}$ **is a constant based on the hardware design**

$$a_f = \frac{A V_f}{\sqrt{\Delta x^2 + \Delta y^2}}$$

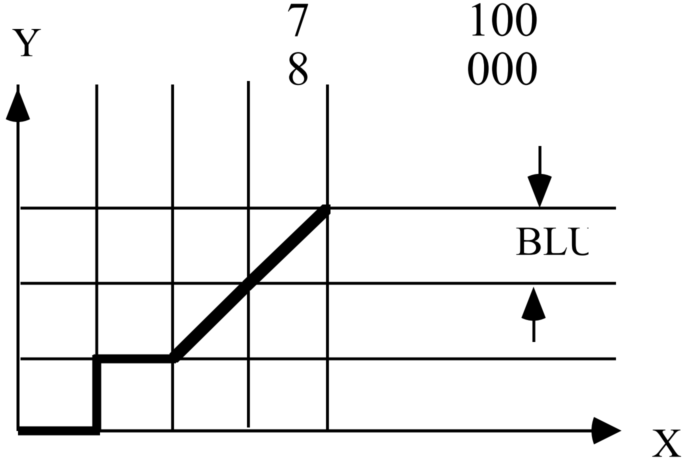
**This is called inversed time code.**

$A$ **value is usually 10.**

# EXAMPLE

**N = 3**
**dX = 4 BLU**
**dY = 3 BLU**

| clock | X | X counter | Y | Y counter |
|---|---|---|---|---|
| 0 | 000 | 0 | 000 | |
| | 0 | | | |
| 1 | 100 | 0 | 011 | 0 |
| 2 | 000 | 1 | 110 | 0 |
| 3 | 100 | 1 | 001 | 1 |
| 4 | 000 | 2 | 100 | 1 |
| 5 | 100 | 2 | 111 | 1 |
| 6 | 000 | 3 | 010 | 2 |
| 7 | 100 | 3 | 101 | 2 |
| 8 | 000 | 4 | 000 | 3 |




**Speed controlled by the clock rate.**

# CIRCULAR INTERPOLATOR

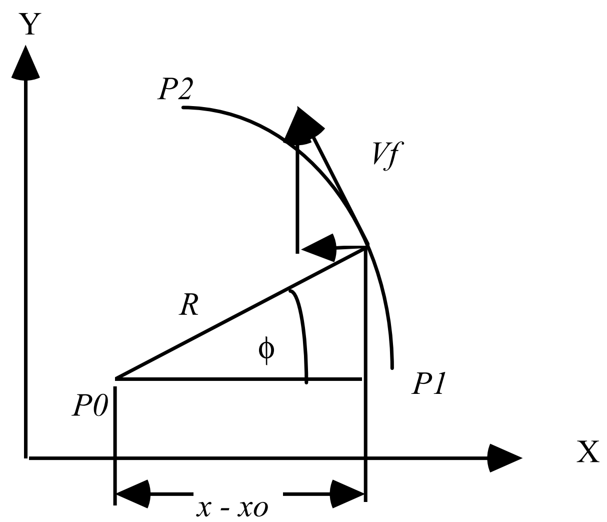



Figure 9.28. A circular arc

$$V_f = R\,\frac{d\phi}{dt}$$

$$x = R\cos\phi + x_0$$

$$y = R\sin\phi + y_0$$

$$R\cos\phi = x - x_0$$

$$R\sin\phi = y - y_0$$

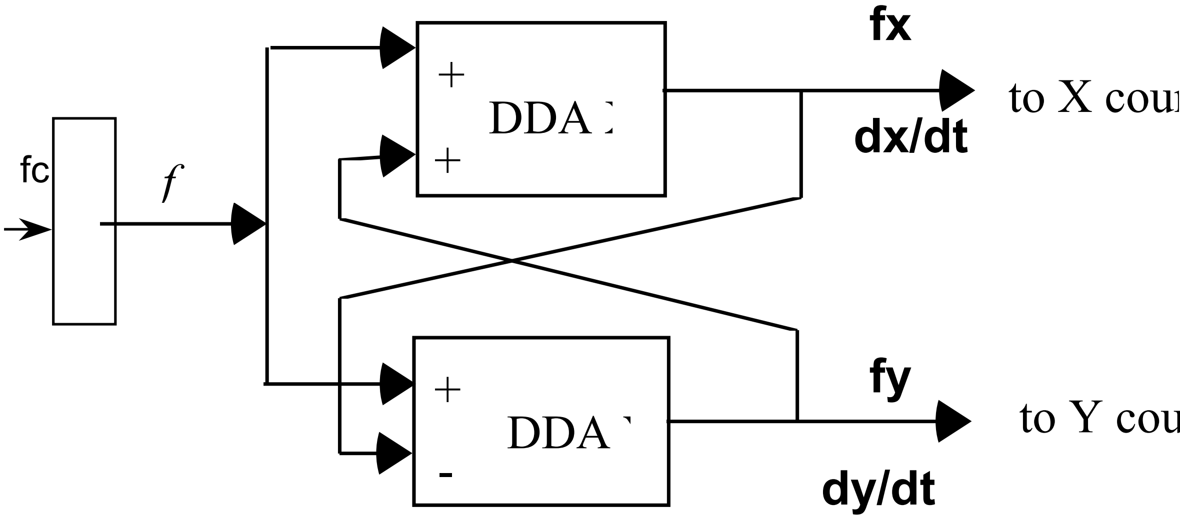
$$\frac{dx}{dt} = -R\sin\phi\,\frac{d\phi}{dt}$$

$$= -(y - y_0)\,\frac{d\phi}{dt}$$

$$\frac{dy}{dt} = R\cos\phi\,\frac{d\phi}{dt}$$

$$= (x - x_0)\,\frac{d\phi}{dt}$$

# CIRCULAR INTERPOLATOR (continue




$$\frac{d^2x}{dt^2} = -R\ \cos\phi\ \frac{d\phi}{dt}\frac{d\phi}{dt}$$

$$= -\frac{dy}{dt}\frac{d\phi}{dt}$$

$$\frac{d^2y}{dt^2} = -R\ \sin\phi\ \frac{d\phi}{dt}\frac{d\phi}{dt}$$

$$= \frac{dx}{dt}\frac{d\phi}{dt}$$

$$a_f = \frac{V_f}{R}\ \frac{2^{(N_f+N)}}{f_c}$$

$$= \frac{10\ V_f}{R}$$

# Future Controllers

- Open architecture
  - Standard hardware platform, plug-and-play
  - Modular software, custom features