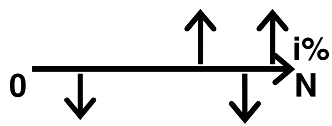
TIME VALUE OF MONEY

- a simple guide -



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limited edition

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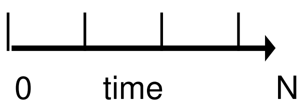
1 - INTRODUCTION

Time value of money has a base in the concepts of monetary flow, time and financial equivalence. It deals with the relation between financial values and the time that such values are associated.
Monetary flow is characterized by putting together, distributed on a time scale, inputs and outputs of monetary values, as many as there are, representing events and their financial dimensions. This is to say that payments and receipts are distributed throughout time, which cannot be summed, subtracted, multiplied or divided, without using resources which compensate for the distances between such values in the time scale in which they are to be found.
With respect to TIME, it may be stated that there is no isolated money value, but always related to a determined time or period. A nominal monetary value today is different from the same nominal value two years from now. Example: Would it be the same in economic terms, to have $1,000 today, comparing with having $1,000 five years from now? Or even, what is the value that a person is prepared to pay today, to receive $10,000, 20 years from now?



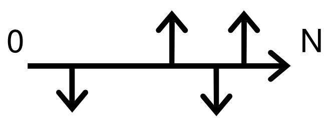
On the above time scale is considered the moment zero (0), which may be the start date of a project or an initial financial application, as well as the final time (N), when the project or the application finishes. It deals with a sequence of equal periods, repeated N times. The unit of measurement may be any unit of time, like seconds, minutes, hours, days, weeks, months, years, decades, etc. The most common units are months and years in financial calculations.

By financial equivalence it is understood that what flows with different values, with an equal number of periods or not, may have the same equivalent value, that is, may have the same FINANCIAL VALUE.

Another concept, no less important, is that of INTEREST, which represents a portion, in value, referent to the use of a monetary resource displaced in time, remunerating this movement. The INTEREST RATE corresponds to the percentage factor that, applied to the monetary value in question, results in the portion of interest. Interest comes from capital income, or from loans or business investments.

Capital is the set of goods available to the economic system for use in the productive capacity of nature (energy, land, raw materials) and human labor, translated into money, which allows the different kinds of capital in comparable systems to be equalized.

For the design of this publication, the following convention will be used:



- ✓ horizontal axis = time axis, from zero (time now) to N (final time)
- ✓ up arrows = value inputs, positive algebraic values, incomes
- ✓ down arrows = value outputs, negative algebraic values, expenses

- ✓ P = capital, current value, value now, value at time zero, principal
- ✓ S = future value, final value
- ✓ R = element of the uniform series, remuneration per period
- ✓ N = number of periods, time
- ✓ i = rate of interest per period of capitalization (decimal value)
- ✓ i% = rate of interest per period of capitalization (percentage value)
- ✓ j = rate of interest accumulated in a total period N (decimal value)
- ✓ j% = rate of interest accumulated in a total period N (percentage value)
- ✓ ^ power of (example 2^3 = 2x2x2=8)

The inputs and outputs of value, income and expense, are always considered at the end of each cited period (end of the day, week, month, year etc.), unless stated otherwise.
The capitals may be of the instantaneous type (capital shown vertically) or non-instantaneous (capital shown horizontally). The first starts at a determined point and the calculations are made by discrete computation of the interest. The second occurs when the income or payment is distributed throughout the period, like the gains of a retail store every day, while the unit of time for calculations is, for example, monthly. In this, the computation of interest is of the continuous type. These two models are associated to the difference of how measurements are done and how the phenomenon occurs.

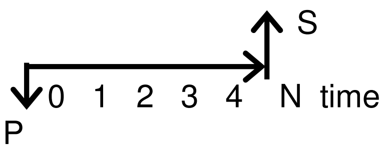
2 – SINGLE PAYMENT

When the financial flow is comprised of only one value input and output, it is said to be dealing with a single payment system.

There are four variables, which are: P, S, N, I. The variables are to be found related by a mathematical expression in such a manner that, provided three of them are known, the missing one may be calculated. The following figure represents a system of single payments schematically:

S = P x (1 + i) ^ N

i = interest rate / period



2.1-Future Value

What is the value that is obtained from the application of a capital $1,000 for 12 months, when the interest rate is equal to 0.1% per month?

S = P (1 + i / 100) ^ N

$$S = 1{,}000 \times (1 + 0.1/100) ^ {12} = 1{,}012.06$$

2.2 – Current value

What is the value that should be invested today to receive $1,000 12 months from now, knowing that the interest rate is 0.2% per month?

$$P = \frac{S}{(1 + i) ^ N}$$

$$P = \frac{1000}{(1 + 0.2/100) ^ {12}} = 976.30$$

2.3 – Capitalization Time

What is the time necessary to obtain a value of $1,100, if $1,000 was applied with an interest rate of 0.3% per month?

$$N = \frac{LOG(S/P)}{LOG(1 + i/100)}$$

$$N = \frac{LOG(1{,}100 / 1{,}000)}{LOG(1 + 0.3/100)} =$$

= 31.8 months

2.4 –Interest Rate

What is the interest rate from an investment where $1,000 is applied today and double this is obtained after 12 months?

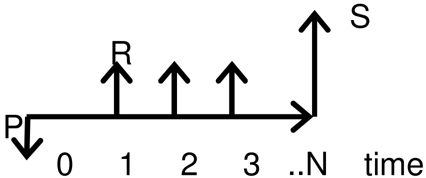
i = (S / P) ^ (1/N) - 1

i = (2000 / 1000) ^ (1/12) - 1 =
0.059 = 5.9% per month

<u>3 – UNIFORM SERIES</u>

Occurs when the financial flow is comprised of a series of equal values, distributed throughout time when a uniform distribution and the time interval between the consecutive values are constant.
The following figure represents schematically, a system with a uniform series of payments:

i = interest rate / period



There are four variables, which are: P (or S), N, I, R. The variables are to be found related by mathematical expressions in such a manner that, as in 2-Single Payment above, if three are known, the fourth one can be calculated.

$$S = \frac{R \times ((1 + i)^N - 1)}{i}$$

$$P = \frac{R \times (1 - (1 + i)^{(-N)})}{i}$$

3.1 – Capital Formation

If \$200 was applied monthly with an interest rate of 0.1% per month, what would be the value obtained at the end of one year?

$$S = \frac{R \times ((1 + i/100)^{N} - 1)}{i/100}$$

$$S = \frac{200 \times ((1 + 0.1/100)^{12} - 1)}{0.1/100}$$

=2413.24

3.2 – Current value

What is the value, in cash, of an object sold in 12 installments of \$20, with an interest rate / period of 0.1% ?

$$P = \frac{R \times (1 - (1 + i/100) ^{(-N)})}{i/100}$$

$$P = \frac{20 \times (1 - (1 + 0.1/100) ^{(-12)})}{0.1/100}$$

$= 238.44$

3.3 – Series Elements

What is the installment to be paid for a loan of $1,000, when the interest rate is 0.5% per month and the time for payment is 12 months?

$$R = \frac{P \times i / 100}{1 - (1 + i / 100) ^{(-N)}}$$

$$R = \frac{1000 \times 0.5 / 100}{1 - (1 + 0.5 / 100)^{(-12)}}$$

= 86.06

3.4 – Time Series

What is the time necessary to pay for a loan of $1,000, at a periodic interest rate of 0.1% per month, with an installment of $200 / month?

$$N = \frac{LOG (R / (R - P \times i / 100))}{LOG (1 + i / 100)}$$

$$N = \frac{LOG(200/(200 - 1{,}000 \times 0.1 / 100))}{LOG (1 + 0.1 / 100)}$$

= 5.01

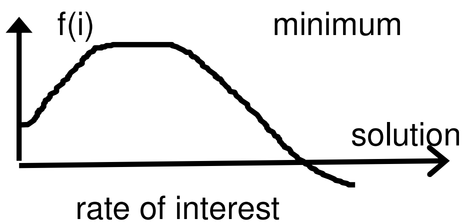
Note: In this calculation it is necessary that R > P x i, that is, the product of the capital, times the interest rate is less than the element value of the series

3.5 – Series Interest rate

What is the interest rate for a loan of $1,000, for 12 installments of $90 per month?

$$f(i) = R \times (1 - (1 + i)^{(-N)}) - P \times i$$

The answer is the value of i which cancels f (i) out



The successive approximations method allows the value of i to be adjusted gradually, until that which is found cancels out the implicit function shown.

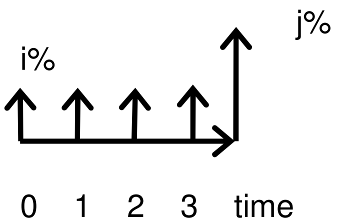
i	* f(i)	notes
1.00	0.1296	First try
2.00	-0.9644	Second try
1.50	-0.2749	It is between 1.0 and 1.5
1.30	-0.0778	Continue getting closer
1.25	-0.0358	....
1.22	-0.0120	....
1.21	-0.0043	Acceptable value

* f (i) = 200 x (1 - (1 + i / 100) ^(-12)) -1,000 x i / 100

The value of i = 1.21 leads the function f(i) to acceptable reduced values. Simple programs or electronic spreadsheets allow these calculations to be done fast and accurately.

4 – INTEREST RATE

The kinds of interest in financial systems should be explicitly stated, if capitalized from period to period and if applied at the start or the end of each accounting period.



$$j\% = ((1 + i\% / 100)^{(N)} - 1) \times 100$$

4.1 – Accumulated Rate

Determine the interest rate accumulated in 12 months for a monthly rate of 0.3%.

$$j\% = ((1 + i\% / 100)^{(N)} - 1) \times 100$$

$$j\% = ((1 + 0.3/100)^{12} - 1) \times 100 = 3.66$$

4.2 – Rate per Period

If the accumulated rate of interest in a year is 4.4%, what is the monthly rate?

$$i\% = \left(\left(J/100 + 1\right)^{(1/N)} - 1\right) \times 100$$

$$i\% = \left(\left(4.4/100 + 1\right)^{(1/12)} - 1\right) \times 100 = 0.35\%$$

4.3 – Series Time

What is the time necessary to equal an accumulated rate of interest of 5.8% to an interest rate per period of 1%?

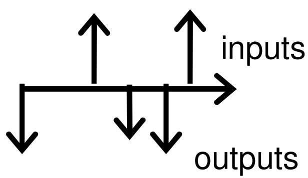
$$N = \frac{LOG(J/100 + 1)}{LOG(i/100 + 1)}$$

$$N = \frac{LOG(5,8/100 + 1)}{LOG(1/100 + 1)}$$

= 5.66 months

5 – NON-UNIFORM SERIES

The financial flow is not always a uniform series of payments,that is, with equal values distributed over identical time intervals.
In the most common market situations, there can be found non-uniform series, or better yet, with different values placed at distinct times (irregular intervals).



5.1 – Equivalent in Time

Given the following flow, what is its equivalent value in the tenth time period with an interest rate of 1% per period?

Time	1	2	4	7	10
value	10	20	10	30	5

time	value	Equivalence at time 10
1	10	10.94

2	20	21.66
4	10	10.62
7	30	30.91
10	5	5.00

The indicated solution is comprised of the transfer of each installment to its equivalent in period 10, by means of:

P (10) = P x (1 + 1 / 100) ^ (10-t)

(P being the value of the installment and t the indexed time of this installment)

Each value is displaced, through the difference in the number of periods between where it is found versus where it should be, based on the interest rate. When all the installments are indexed to the desired period, they can be summed algebraically, resulting in just one value that is equivalent to the series given previously. The value of 79.12, in the example, is equivalent to the series given.

In another flow, with inputs and outputs of values (identified by means of the signs) as shown, what would be its equivalent value, given the same rate and the same time destination 10?

Time	1	2	4	7	10
value	10	-20	-10	30	-5

P(10) = P x (1 + 10 / 100) ^ (10-t)

time	Value	Equivalence at time 10
1	10	10.94
2	-20	-21.66
4	-10	-10.62
7	30	30.91
10	-5	-5.00
	sum=	4.57

The value 4.57 is equivalent to the series given.

5.2 – Interest rate of a Series

Given a non-uniform series and a present value (equivalent value at time zero) that is equal to the series given, the interest rate which balances the system may be calculated.

Given the following series:

Time:	1	3	4	7
value:	30	40	50	60

And the present value equal to $150, what is the interest rate?

It is important to state that, in order to get a solution, the algebraic sum of the values (independent of interest or time) must be greater than the principal (present value).
30 + 40 + 50 + 60= 180 must be greater than 150

Time	1	3	4	7	sum
value	30	40	50	60	180
i=3	29	36	44	48	158
i=5	28	34	41	42	146
i=4	28	35	42	45	152
i=4.3	28	35	42	44	151
i=4.4	28	35	42	44	150

Therefore, when i is equal to approximately 4.4%, the function approximates the value 150 and reaches its objective.