Lista de Exercícios de CVE
Capítulo 2: Trigonometria na circunferência

Trigonometria na circunferência.

Exemplo 1: transforme 50^0 em radianos.

Solução: para resolver esse problema, lembremo-nos que 180^0 corresponde a π radianos. Usando a regra de 3 seguinte, obtemos o resultado.

$$\begin{array}{l} \pi \text{ —— } 180^0 \\ x \text{ —— } 50^0 \end{array}$$

Temos, então,

$$180x = 50\pi \Rightarrow x = \frac{50\pi}{180} \Rightarrow x = \frac{5\pi}{18}.$$

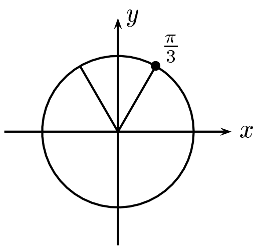
Portanto, $50^0 = \frac{5\pi}{18} rad$.

E1) Transforme as medidas dos ângulos abaixo em radianos:

a) 30^0, b) 45^0, c) 60^0, d) 90^0, e) 270^0, f) 360^0.

Exemplo 2: indique no ciclo trigonométrico o ângulo $\frac{\pi}{3}$.

Solução: este ângulo está em uma posição que divide o semicírculo que vai de 0 a π em 3 pedaços:



E2) Indique no ciclo trigonométrico os seguintes ângulos:

a) $\frac{\pi}{6}$, b) $\frac{\pi}{4}$, c) $\frac{\pi}{2}$, d) π, e) $\frac{3\pi}{2}$, f) 2π, g) $\frac{3\pi}{4}$, h) $\frac{5\pi}{6}$, i) $\frac{5\pi}{3}$.

Exemplo 3: escreva o seno e o cosseno do ângulo $\frac{\pi}{3}$.

Solução: temos:

$$\operatorname{sen}\frac{\pi}{3} = \frac{\sqrt{3}}{2}, \quad \cos\frac{\pi}{3} = \frac{1}{2}.$$

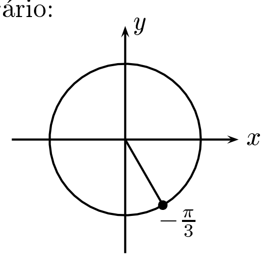
E3) Escreva os senos e cossenos dos seguintes ângulos:

a) $\frac{\pi}{6}$, b) $\frac{\pi}{4}$, c) $\frac{\pi}{3}$, d) $\frac{\pi}{2}$, d) π, e) $\frac{3\pi}{2}$, f) 2π, g) $\frac{3\pi}{4}$, h) $\frac{5\pi}{6}$, i) $\frac{5\pi}{3}$.

Ângulos negativos

Exemplo 4: indique no ciclo trigonométrico o ângulo $-\frac{\pi}{3}$.

Solução: este ângulo está em uma posição que divide o semicírculo que vai de 0 a π em 3 pedaços, só que partindo da origem e seguindo o sentido horário:



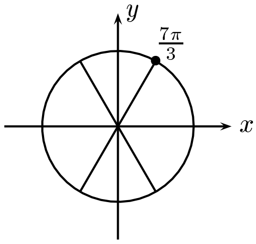
E4) Indique no ciclo trigonométrico os seguintes ângulos:

a) $-\frac{\pi}{6}$, b) $-\frac{\pi}{4}$, c) $-\frac{\pi}{2}$, d) $-\pi$, e) $-\frac{3\pi}{2}$, f) -2π, g) $-\frac{3\pi}{4}$, h) $-\frac{5\pi}{6}$, i) $-\frac{5\pi}{3}$.

Ângulos maiores que 2π e menores que -2π.

Exemplo 5: indique no ciclo trigonométrico o ângulo $\frac{7\pi}{3}$.

Solução: contamos sete vezes o ângulo $\frac{\pi}{3}$, partindo da origem em sentido anti-horário:



E5) Indique no ciclo trigonométrico os seguintes ângulos:

a) $\frac{15\pi}{6}$, b) $\frac{13\pi}{4}$, c) $\frac{6\pi}{2}$, d) 7π, e) $\frac{11\pi}{3}$, f) $-\frac{15\pi}{4}$, g) $-\frac{15\pi}{6}$, h) $-\frac{7\pi}{3}$.

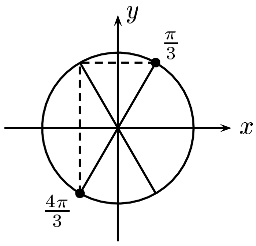
Redução ao primeiro quadrante.

Exemplo 6: escreva o seno e o cosseno do ângulo $\frac{4\pi}{3}$.

Solução:

primeiro, indicamos a posição do ângulo no ciclo trigonométrico. Depois, analisamos sua posição com relação ao ângulo $\frac{\pi}{3}$. Daí, podemos ver que

$$\begin{array}{l} \operatorname{sen}\frac{4\pi}{3} = -\operatorname{sen}\frac{\pi}{3} = -\frac{\sqrt{3}}{2}, \\ \cos\frac{4\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2} \end{array}.$$



E6) Escreva os senos e cossenos dos seguintes ângulos:

a) $\frac{7\pi}{6}$, b) $\frac{9\pi}{4}$, c) $-\frac{\pi}{6}$, d) $-\frac{\pi}{2}$, e) 3π, f) $\frac{3\pi}{2}$, g) $-\pi$, h) $\frac{7\pi}{4}$, i) $\frac{5\pi}{6}$, j) $\frac{8\pi}{3}$, k) $-\frac{2\pi}{3}$, l) $\frac{11\pi}{4}$.

Cosseno e seno da soma de ângulos.

Exemplo 7: usando a fórmula do seno da soma, calcule $\operatorname{sen}\frac{5\pi}{6} = \operatorname{sen}\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$.

Solução: usando a fórmula fundamental $\operatorname{sen}(a+b) = \operatorname{sen} a.\cos b + \cos a.\operatorname{sen} b$, temos:

$$\operatorname{sen}\frac{5\pi}{6} = \operatorname{sen}\left(\frac{\pi}{2}+\frac{\pi}{3}\right) = \operatorname{sen}\frac{\pi}{2}.\cos\frac{\pi}{3} + \cos\frac{\pi}{2}.\operatorname{sen}\frac{\pi}{3} = 1.\frac{1}{2} + 0.\frac{\sqrt{3}}{2} = \frac{1}{2}.$$

E7) Usando as fórmulas do seno da soma e do cosseno da soma, calcule os senos e cossenos abaixo:

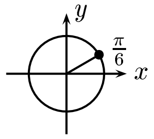
a) $\operatorname{sen}\frac{5\pi}{12} = \operatorname{sen}\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$, b) $\operatorname{sen}\frac{11\pi}{12} = \operatorname{sen}\left(\frac{3\pi}{4}+\frac{\pi}{6}\right)$, c) $\operatorname{sen}\frac{\pi}{12} = \operatorname{sen}\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$,

d) $\operatorname{sen}\left(\frac{-7\pi}{12}\right) = \operatorname{sen}\left(\frac{\pi}{4}-\frac{\pi}{3}\right)$, e) $\cos\frac{5\pi}{12} = \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$, f) $\cos\frac{11\pi}{12} = \cos\left(\frac{3\pi}{4}+\frac{\pi}{6}\right)$,

g) $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$, h) $\cos\left(\frac{-7\pi}{12}\right) = \cos\left(\frac{\pi}{4}-\frac{\pi}{3}\right)$.

Respostas

E1) a) $\frac{\pi}{6}$; b) $\frac{\pi}{4}$; c) $\frac{\pi}{3}$; d) $\frac{\pi}{2}$; e) $\frac{3\pi}{2}$; f) 2π.

E2) a)


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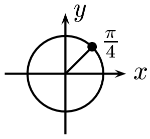

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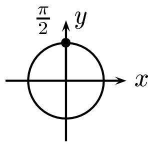

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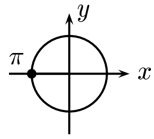

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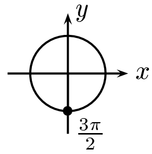

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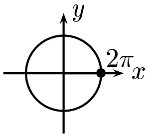
f)


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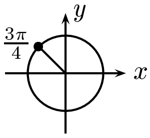

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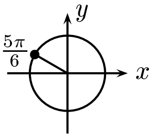

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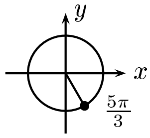

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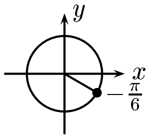
E3) a) $\operatorname{sen}\frac{\pi}{6} = \frac{1}{2}$, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$; b) $\operatorname{sen}\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$; c) $\operatorname{sen}\frac{\pi}{2} = 1$, $\cos\frac{\pi}{2} = 0$; d) $\operatorname{sen}\pi = 0$, $\cos\pi = -1$;
e) $\operatorname{sen}\frac{3\pi}{2} = -1$, $\cos\frac{3\pi}{2} = 0$; f) $\operatorname{sen}2\pi = 0$, $\cos 2\pi = 0$; g) $\operatorname{sen}\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$, $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$; h) $\operatorname{sen}\frac{5\pi}{6} = \frac{1}{2}$, $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$;
i) $\operatorname{sen}\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$, $\cos\frac{5\pi}{3} = \frac{1}{2}$.

E4) a)


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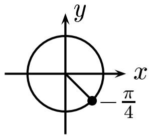

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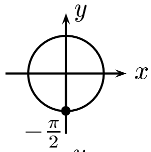

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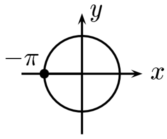

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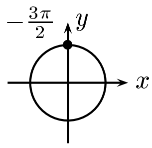

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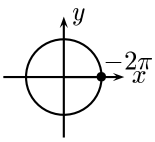

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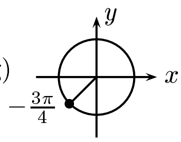

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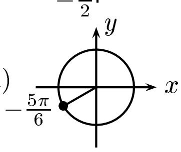

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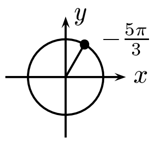

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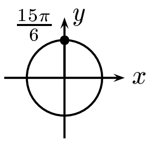

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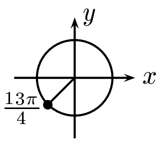

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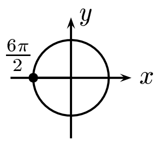

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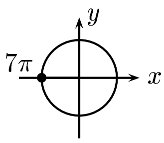

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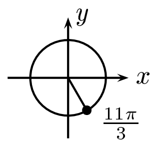

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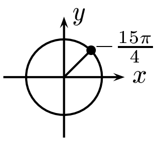

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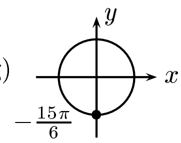

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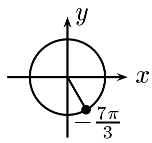

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E6) a) $\operatorname{sen}\frac{7\pi}{6} = -\frac{1}{2}$, $\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$; b) $\operatorname{sen}\frac{9\pi}{4} = \frac{1}{\sqrt{2}}$, $\cos\frac{9\pi}{4} = \frac{1}{\sqrt{2}}$; c) $\operatorname{sen}\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$, $\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$;
d) $\operatorname{sen}\left(-\frac{\pi}{2}\right) = -1$, $\cos\left(-\frac{\pi}{2}\right) = 0$; e) $\operatorname{sen}3\pi = 0$, $\cos 3\pi = -1$; f) $\operatorname{sen}\frac{3\pi}{2} = -1$, $\cos\frac{3\pi}{2} = 0$;
g) $\operatorname{sen}(-\pi) = 0$, $\cos(-\pi) = -1$; h) $\operatorname{sen}\frac{7\pi}{4} = -\frac{1}{\sqrt{2}}$, $\cos\frac{7\pi}{4} = \frac{1}{\sqrt{2}}$, i) $\operatorname{sen}\frac{5\pi}{6} = \frac{1}{2}$, $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$;
j) $\operatorname{sen}\frac{8\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{8\pi}{3} = -\frac{1}{2}$; k) $\operatorname{sen}\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$, $\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$, l) $\operatorname{sen}\frac{11\pi}{4} = \frac{1}{\sqrt{2}}$, $\cos\frac{11\pi}{4} = -\frac{1}{\sqrt{2}}$.

E7) a) $\frac{1+\sqrt{3}}{2\sqrt{2}}$, b) $\frac{-1+\sqrt{3}}{2\sqrt{2}}$, c) $\frac{-1+\sqrt{3}}{2\sqrt{2}}$, d) $\frac{1-\sqrt{3}}{2\sqrt{2}}$, e) $\frac{-1+\sqrt{3}}{2\sqrt{2}}$, f) $\frac{-1-\sqrt{3}}{2\sqrt{2}}$, g) $\frac{1+\sqrt{3}}{2\sqrt{2}}$, d) $\frac{1+\sqrt{3}}{2\sqrt{2}}$.