Lista de Exercícios de GA
Capítulo 2: Curvas no plano: equações paramétricas

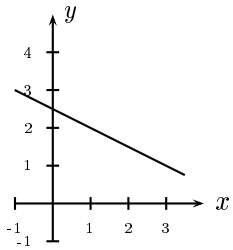
Equações paramétricas.

Exemplo 1: esboce a reta dada pelas equações paramétricas $\begin{cases} x = 1 + 2t, \\ y = 2 - t. \end{cases}$

Solução:

$t = 0: \begin{cases} x = 1 + 2.0 = 1 + 0 = 1, \\ y = 2 - 0 = 2. \end{cases}$

$t = 1: \begin{cases} x = 1 + 2.1 = 1 + 2 = 3, \\ y = 2 - 1 = 1. \end{cases}$



E1) Esboce as retas dadas pelas seguintes equações paramétricas:

a) $\begin{cases} x = t, \\ y = t; \end{cases}$ b) $\begin{cases} x = 1 + t, \\ y = 2 - t; \end{cases}$ c) $\begin{cases} x = 2 - 3t, \\ y = 1 + t; \end{cases}$ d) $\begin{cases} x = t, \\ y = 1 + 2t; \end{cases}$ e) $\begin{cases} x = 2 + 3t, \\ y = 1 - 2t. \end{cases}$

Exemplo 2: esboce a parábola dada pelas equações paramétricas $\begin{cases} x = 1 + 2t, \\ y = 2 - t^2. \end{cases}$.

Solução:

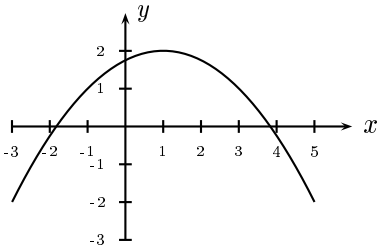
$t = -2: \begin{cases} x = 1 + 2.(-2) = 1 - 4 = -3, \\ y = 2 - (-2)^2 = 2 - 4 = -2. \end{cases}$

$t = -1: \begin{cases} x = 1 + 2.(-1) = 1 - 2 = -1, \\ y = 2 - (-1)^2 = 2 - 1 = 1. \end{cases}$

$t = 0: \begin{cases} x = 1 + 2.0 = 1 + 0 = 1, \\ y = 2 - 0^2 = 2. \end{cases}$

$t = 1: \begin{cases} x = 1 + 2.1 = 1 + 2 = 3, \\ y = 2 - 1^2 = 2 - 1 = 1. \end{cases}$

$t = 2: \begin{cases} x = 1 + 2.2 = 1 + 4 = 5, \\ y = 2 - 2^2 = 2 - 4 = -2. \end{cases}$



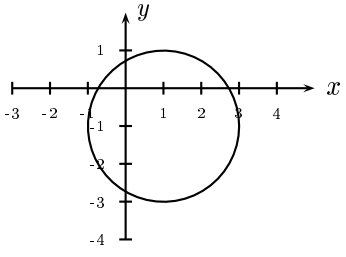
E2) Esboce as parábolas dadas pelas seguintes equações paramétricas:

a) $\begin{cases} x = t, \\ y = t^2; \end{cases}$ b) $\begin{cases} x = t^2, \\ y = t; \end{cases}$ c) $\begin{cases} x = 2 - t^2, \\ y = 1 + t; \end{cases}$ d) $\begin{cases} x = 1 + 2t, \\ y = 1 + t^2; \end{cases}$ e) $\begin{cases} x = 2 + t, \\ y = 3 - t^2; \end{cases}$

f) $\begin{cases} x = 1 + t^2, \\ y = 1 - 2t + t^2; \end{cases}$ g) $\begin{cases} x = 1 + t - 2t^2, \\ y = -3 + t + t^2. \end{cases}$

Exemplo 3: esboce a circunferência dada pelas equações paramétricas $\begin{cases} x = 1 + 2\cos t, \\ y = -1 + 2\,\mathrm{sen}\, t. \end{cases}$.

Solução: esta é uma circunferência de raio 2 centrada em $x = 1$ e $y = -1$.



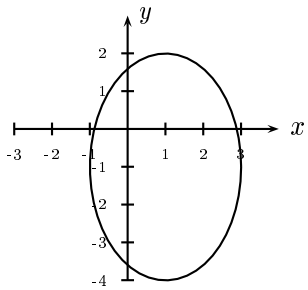
E3) Esboce as circunferências dadas pelas seguintes equações paramétricas:

a) $\begin{cases} x = \cos t, \\ y = \operatorname{sen} t; \end{cases}$ b) $\begin{cases} x = \operatorname{sen} t, \\ y = \cos t; \end{cases}$ c) $\begin{cases} x = 2\cos t, \\ y = 2\operatorname{sen} t; \end{cases}$ d) $\begin{cases} x = 1 + \cos t, \\ y = 2 + \operatorname{sen} t; \end{cases}$ e) $\begin{cases} x = -2 + 2\cos t, \\ y = 1 + 2\operatorname{sen} t; \end{cases}$

f) $\begin{cases} x = \cos t + \operatorname{sen} t, \\ y = \cos t - \operatorname{sen} t; \end{cases}$ g) $\begin{cases} x = 2 + \cos t + \operatorname{sen} t, \\ y = 1 + \operatorname{sen} t - \operatorname{sen} t; \end{cases}$ e) $\begin{cases} x = 1 + \cos t - 3\operatorname{sen} t, \\ y = 2 + 3\cos t + \operatorname{sen} t. \end{cases}$

Exemplo 4: esboce a elipse dada pelas equações paramétricas $\begin{cases} x = 1 + 2\cos t, \\ y = -1 + 3\operatorname{sen} t. \end{cases}$.

Solução: esta é uma elipse de aresta 2 em x e aresta 3 em y, centrada em $x = 1$ e $y = -1$.



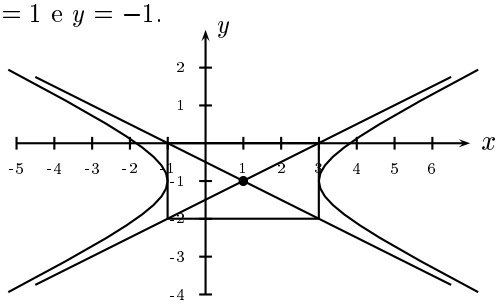
E4) Esboce as elipses dadas pelas seguintes equações paramétricas:

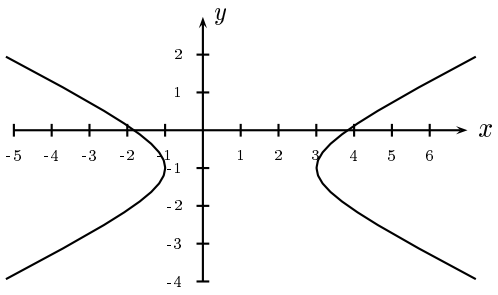
a) $\begin{cases} x = 3\cos t, \\ y = 2\operatorname{sen} t; \end{cases}$ b) $\begin{cases} x = \cos t, \\ y = 3\operatorname{sen} t; \end{cases}$ c) $\begin{cases} x = 1 + 2\cos t, \\ y = -2 + 3\operatorname{sen} t; \end{cases}$ d) $\begin{cases} x = -3 + 2\cos t, \\ y = 2 + \operatorname{sen} t; \end{cases}$

e) $\begin{cases} x = 2\cos t + \operatorname{sen} t, \\ y = \cos t - 3\operatorname{sen} t; \end{cases}$ f) $\begin{cases} x = 1 + \cos t - 3\operatorname{sen} t, \\ y = 2 - 3\cos t + \operatorname{sen} t. \end{cases}$

Exemplo 5: esboce a hipérbole dada pelas equações paramétricas $\begin{cases} x = 1 \pm 2\cosh t, \\ y = -1 \pm \operatorname{senh} t. \end{cases}$.

Solução: esta é uma hipérbole cujas assíntotas seguem as diagonais do retângulo de base 2 e altura 1, centrada em $x = 1$ e $y = -1$.





E5) Esboce as hipérboles dadas pelas seguintes equações paramétricas:

a) $\begin{cases} x = \pm 3\cosh t, \\ y = \pm 2\operatorname{senh} t; \end{cases}$ b) $\begin{cases} x = \pm\cosh t, \\ y = \pm 3\operatorname{senh} t; \end{cases}$ c) $\begin{cases} x = 1 \pm 2\cosh t, \\ y = -2 \pm 3\operatorname{senh} t; \end{cases}$ d) $\begin{cases} x = -3 \pm 2\cosh t, \\ y = 2 \pm \operatorname{senh} t; \end{cases}$

e) $\begin{cases} x = \pm 3\cosh t, \\ y = \pm\operatorname{senh} t; \end{cases}$ f) $\begin{cases} x = 2 \pm 2\cosh t, \\ y = -1 \pm 3\operatorname{senh} t; \end{cases}$ g) $\begin{cases} x = \pm(2\cosh t + \operatorname{senh} t), \\ y = \pm(\cosh t - 3\operatorname{senh} t). \end{cases}$

Respostas

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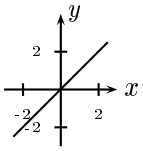
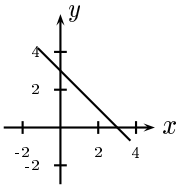
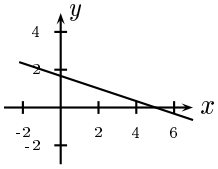
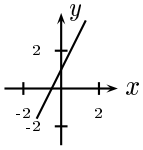
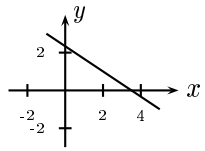
a) 

b) 

c) 

d) 

e) 

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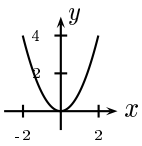
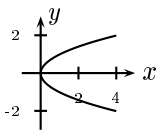
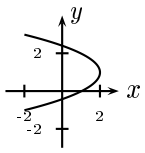
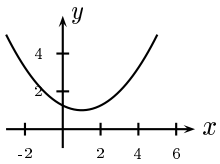
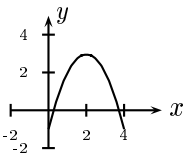
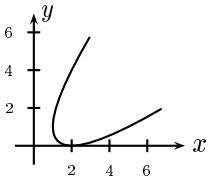
a) 

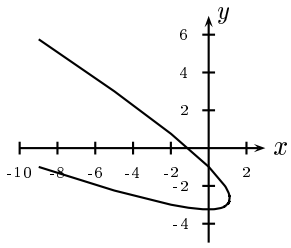
b) 

c) 

d) 

e) 

f) 

g) 

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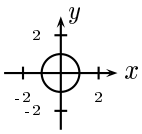
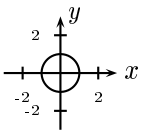
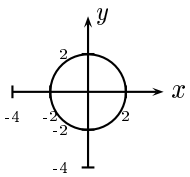
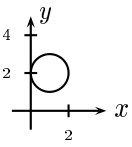
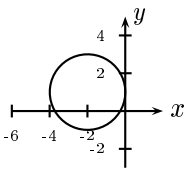
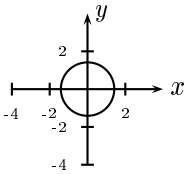
a) 

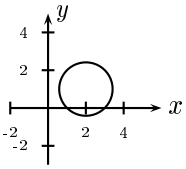
b) 

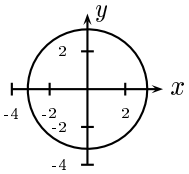
c) 

d) 

e) 

f) 

g) 

h) 

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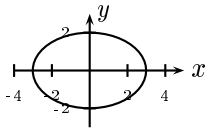
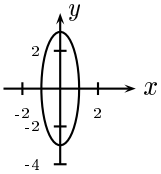
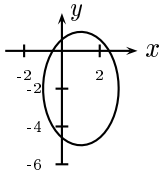
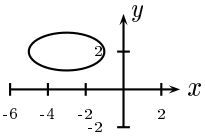
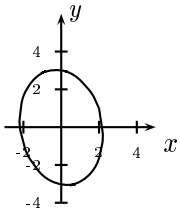
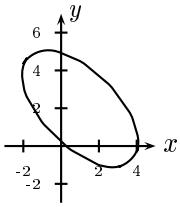
a) 

b) 

c) 

d) 

e) 

f) 

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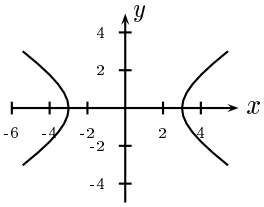
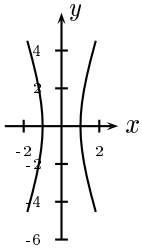
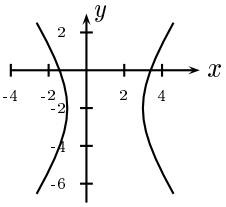
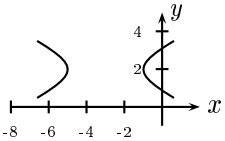
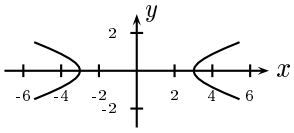
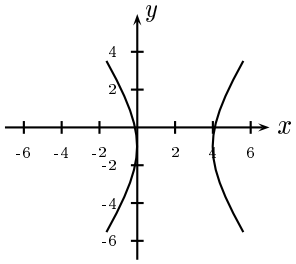
a) 

b) 

c) 

d) 

e) 

f) 

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