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Fading Resistance of Orthogonal Space-Time Block Codes Under Spatial Correlation

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Abstract — **In this paper, impact on the bit-error rate performance of orthogonal space-time block codes (O-STBC) due to spatial correlation is investigated. An analytic model for spatial correlation is used which fully accounts for: i) antenna placement and separation, and ii) scattering distributions (Isotropic, Uniform-limited, etc.). We show that the impact of the space is limited on the bit-error rate performance of O-STBC, that is most of the error rate improvement is due to the 'time-coding' than to 'space-coding'. Further, we investigate how the non-isotropic parameters of an azimuth power distribution, including the angular spread and the mean angle of arrival (AOA) of an impinging signal effect the bit-error rate performance of O-STBC. A rule of thumb for the antenna separation is proposed, where the performance of the O-STBC is optimal under a given scattering environment.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) antenna system together with space-time coding can significantly improve the performance of a wireless communication system by exploring the spatial and temporal diversities of the system. In general, the presence of spatial fading correlation between antenna elements will effect the performance of any space-time coding scheme. However, the O-STBC has an inherent protection against information loss due to the spatially correlated fading [1], [2]. This motivates the investigation of the degree of fading resistance provided by O-STBC when spatial correlation is present.

Spatial correlation has two sources: i) antenna placement (particularly antenna separation) and ii) scattering distribution (Isotropic and Non-Isotropic). This paper investigates the impact of both sources on the performance of O-STBC. It is well known that larger the antenna separation, lesser the spatial correlation between antenna elements. Therefore we pay special attention to the antenna separation distance and examine the effects of it on the performance of O-STBC, under a non-isotropic scattering environment. We also suggest a rule of thumb for the minimal separation between antennas where most of the performance gains expected from O-STBC are achieved.

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II. Channel Model and Spatial Correlation Matrices

Consider the MIMO antenna system in 2-D space shown in Fig.1, where the transmitter consist of n_T transmit antennas and the receiver consist of n_R receive antennas. Assume that the channel gains between transmitter and receiver antennas undergo flat fading and also assume that scatterers are distributed in the farfield from the receiver and the transmitter antenna apertures. The channel gain between the t^{th} transmitter antenna and the r^{th} receiver antenna can be defined as [3]

$$h_{rt} = \int_{\Omega_t}\int_{\Omega_r} g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}}) e^{-ik\boldsymbol{y_r}\cdot\hat{\boldsymbol{\varphi}}} e^{ik\boldsymbol{x_t}\cdot\hat{\boldsymbol{\phi}}} d\hat{\boldsymbol{\phi}} d\hat{\boldsymbol{\varphi}}, \quad (1)$$

where $g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})$ is the effective random scattering gain function for a signal leaving from the transmitter scattering free aperture at a direction $\hat{\boldsymbol{\phi}} \equiv (1,\phi)$ and entering the receiver scattering free aperture from a direction $\hat{\boldsymbol{\varphi}} \equiv (1,\varphi)$, $\boldsymbol{x_t}$ is the location of the t^{th} transmitter antenna and $\boldsymbol{y_r}$ is the location of the r^{th} receiver antenna relative to the transmitter and receiver array origins respectively, $k = 2\pi/\lambda$ is the wave number with λ being the wave length and $i = \sqrt{-1}$. This channel representation allows us to model the spatial channel for any physical antenna configuration and also for any general scattering distribution as $\boldsymbol{x_t}$ and $\boldsymbol{y_r}$ tell us about the physical antenna configuration and $g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})$ represents the surrounding random scattering environment, which could be modelled as a random function. We assume that the channel gains are normalized such that $E\left\{|h_{rt}|^2\right\} = 1$ and hence the scattering gains $g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})$ are normalized such that $\int_{\Omega_t}\int_{\Omega_r} E\left\{|g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})|^2\right\} d\hat{\boldsymbol{\phi}} d\hat{\boldsymbol{\varphi}} = 1$, where $E\{\cdot\}$ denotes mathematical expectation.

Channel Correlation Coefficients

The complex channel correlation coefficient between receiver antennas r_1 and r_2 can be defined as

$$\rho^{\text{Rx}}_{r_1 r_2} = E\left\{h_{r_1 t} h^*_{r_2 t}\right\}, \quad \text{for all } t, \quad (2)$$

where $\cdot^*$ is the complex conjugate operator. Note that, this definition is subject to the assumptions made below.

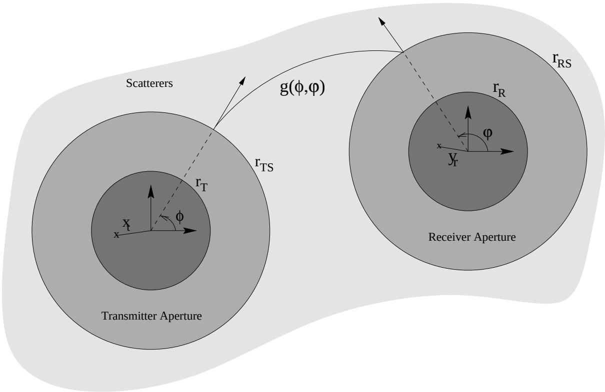

Fig. 1: A general 2-D scattering model for a flat fading MIMO system. r_T and r_R are radii of circular apertures which enclose the transmitter and receiver arrays respectively. The scattering environment is represented by $g(\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\varphi}})$ which gives the effective random complex gain for signals leaving the transmitter aperture from direction $\hat{\boldsymbol{\phi}}$ and arriving at the receiver aperture from direction $\hat{\boldsymbol{\varphi}}$.

- All antenna elements in the receiver and the transmitter antenna arrays have the same polarization and the same radiation pattern identical to each other.
- Correlation between two antenna elements in one antenna array is independent of the antenna element selected from the other antenna array. [4].

Using (2), the receiver channel correlation matrix $\mathbf{R}_{\mathrm{Rx}} \in \mathbb{C}^{n_R \times n_R}$ can be defined as

$$\mathbf{R}_{\mathrm{Rx}} \triangleq \begin{pmatrix} \rho_{11}^{\mathrm{Rx}} & \cdots & \rho_{1n_R}^{\mathrm{Rx}} \\ \vdots & \ddots & \vdots \\ \rho_{n_R1}^{\mathrm{Rx}} & \cdots & \rho_{n_Rn_R}^{\mathrm{Rx}} \end{pmatrix}_{\mathrm{n_R \times n_R}} . \tag{3}$$

A similar definition can be given for the correlation matrix at the transmitter $\mathbf{R}_{\mathrm{Tx}} \in \mathbb{C}^{n_T \times n_T}$ in terms of complex channel correlation coefficients observed at the transmitter, say $\rho_{t_1t_2}^{\mathrm{Tx}}$. Note that the matrices $\mathbf{R}_{\mathrm{Rx}}$ and $\mathbf{R}_{\mathrm{Tx}}$ are Hermitian matrices.

The correlation coefficient between two arbitrary channel gains connecting two input-output points of antennas is given by [4]

$$\rho_{t_2r_2}^{t_1r_1} \triangleq E\left\{h_{r_1t_1} h_{r_2t_2}^*\right\} = \rho_{t_1t_2}^{\mathrm{Tx}} \rho_{r_1r_2}^{\mathrm{Rx}}, \tag{4}$$

provided that $\rho_{r_1r_2}^{\mathrm{Rx}}$ is independent of t and $\rho_{t_1t_2}^{\mathrm{Tx}}$ is independent of r.

This gives that the correlation matrix of the MIMO channel $\mathbf{H}$ as the Kronecker product of the correlation matrices observed at the transmitter and the receiver [5], i.e,

$$\mathbf{R}_{\mathrm{H}} = \mathbf{R}_{\mathrm{Tx}} \otimes \mathbf{R}_{\mathrm{Rx}}, \tag{5}$$

where $\otimes$ denotes the matrix Kronecker product and $\mathbf{R}_{\mathrm{H}} \in \mathbb{C}^{n_Rn_T \times n_Rn_T}$ is a positive definite matrix. Note that this Kronecker product model is a special case where correlation among all transmitter-receiver antenna branches are taken into consideration when defining the correlation matrix of the MIMO channel.

Generation of correlated channel gains

As shown above, the spatial correlation coefficients observed at the receiver and the transmitter antennas can be expressed in terms of random channel gains between transmitter and receiver antennas. Here we briefly outline a method which could be used to generate correlated channel gains h_{rt} from an uncorrelated MIMO channel matrix $\mathbf{A}$. This method was originally proposed in [4] and some modifications[1] to it has been made to suite our work as outlined below.

- Obtain the lower triangular Cholskey matrix $\mathbf{C}$ such that $\mathbf{R}_{\mathrm{H}} = \mathbf{C}\mathbf{C}^T$.
- Generate an independent and identically distributed $n_R \times n_T$ channel matrix $\mathbf{A}$ whose elements are complex gaussian distributed with zero-mean and unit variance.
- Let $\boldsymbol{a} = \mathrm{vec}(\mathbf{A})$ where $\mathrm{vec}(\cdot)$ is the vector operator.
- Correlated channel gains of the MIMO channel $\mathbf{H}$ is given by $\boldsymbol{h} = \mathbf{C}\boldsymbol{a}$, where $\boldsymbol{h} = \mathrm{vec}(\mathbf{H})$.

III. Transmitter/Receiver spatial correlation coefficients as a function of antenna separation

The normalized spatial correlation function between the complex envelopes of two received narrowband signals z_1 and z_2, located at points $\boldsymbol{y_1}$ and $\boldsymbol{y_2}$ respectively, is given by

$$\rho(\boldsymbol{y_2} - \boldsymbol{y_1}) = \frac{E\{z_1 z_2^*\}}{\sqrt{E\{z_1 z_1^*\} E\{z_2 z_2^*\}}}, \tag{6}$$

It was shown in [6] that if the transmitted signal is a narrowband signal $e^{i\omega t}$, then the spatial correlation function $\rho(\boldsymbol{y_2} - \boldsymbol{y_1})$ is given by

$$\rho(\boldsymbol{y_2} - \boldsymbol{y_1}) = \int \mathcal{P}(\hat{\boldsymbol{\phi}}) e^{ik(\boldsymbol{y_2} - \boldsymbol{y_1}) \cdot \hat{\boldsymbol{\phi}}} d\hat{\boldsymbol{\phi}}, \tag{7}$$

where $k = 2\pi/\lambda$ is the wave number and

$$\mathcal{P}(\hat{\boldsymbol{\phi}}) = \frac{E\left\{|g(\hat{\boldsymbol{\phi}})|^2\right\}}{\int E\left\{|g(\hat{\boldsymbol{\phi}})|^2\right\} d\hat{\boldsymbol{\phi}}} \tag{8}$$

is the normalized average power of a signal received from direction $\hat{\boldsymbol{\phi}}$, and $g(\hat{\boldsymbol{\phi}})$ is the complex scattering gain as a function of direction $\hat{\boldsymbol{\phi}}$.

Correlation coefficients observed at the receiver

Using (1) and (2) we may write

$$E\left\{h_{r_1t} h_{r_2t}^*\right\} = \int_4 E\left\{g(\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\varphi}}) g^*(\hat{\boldsymbol{\phi}}', \hat{\boldsymbol{\varphi}}')\right\} e^{-ik(\boldsymbol{y_{r_1}} \cdot \hat{\boldsymbol{\varphi}} - \boldsymbol{y_{r_2}} \cdot \hat{\boldsymbol{\varphi}}')} \times e^{ik\boldsymbol{x_t} \cdot (\hat{\boldsymbol{\phi}} - \hat{\boldsymbol{\varphi}}')} d\hat{\boldsymbol{\phi}} d\hat{\boldsymbol{\varphi}} d\hat{\boldsymbol{\phi}}' d\hat{\boldsymbol{\varphi}}', \tag{9}$$

where $\int_4 \triangleq \int_{\Omega_t} \int_{\Omega_r} \int_{\Omega_t} \int_{\Omega_r}$. Assume that the complex scattering from one direction is independent from another direction for both at the receiver and the transmitter apertures, also assume that scattering environment is zero-mean uncorrelated, then the second-order statistics of the scattering gain function $g(\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\varphi}})$ can be defined as

$$E\left\{g(\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\varphi}}) g^*(\hat{\boldsymbol{\phi}}', \hat{\boldsymbol{\varphi}}')\right\} = G(\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\varphi}}) \delta(\hat{\boldsymbol{\phi}} - \hat{\boldsymbol{\phi}}') \delta(\hat{\boldsymbol{\varphi}} - \hat{\boldsymbol{\varphi}}'), \tag{10}$$

[1] Power shaping between antenna elements is not considered.

where $\delta(\cdot)$ is the Dirac delta function. Substitution of (10) into (9) gives

$$E\{h_{r_1t}h^*_{r_2t}\} = \int_{\Omega_r}\int_{\Omega_t} G(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})e^{ik(\boldsymbol{y_{r_2}}-\boldsymbol{y_{r_1}}).\hat{\boldsymbol{\varphi}}}d\hat{\boldsymbol{\phi}}d\hat{\boldsymbol{\varphi}}$$
$$= \int_{\Omega_r} \mathcal{P}_{\mathrm{Rx}}(\hat{\boldsymbol{\varphi}})e^{ik(\boldsymbol{y_{r_2}}-\boldsymbol{y_{r_1}}).\hat{\boldsymbol{\varphi}}}d\hat{\boldsymbol{\varphi}}, \qquad (11)$$

where $\mathcal{P}_{\mathrm{Rx}}(\hat{\boldsymbol{\varphi}})$ is the normalized average power received from direction $\hat{\boldsymbol{\varphi}}$ with normalization $\int_{\Omega_t}\int_{\Omega_r} E\left\{|g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})|^2\right\}d\hat{\boldsymbol{\phi}}d\hat{\boldsymbol{\varphi}} = 1$ and

$$\mathcal{P}_{\mathrm{Rx}}(\hat{\boldsymbol{\varphi}}) = \frac{\int_{\Omega_t} E\left\{|g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})|^2\right\}d\hat{\boldsymbol{\phi}}}{\int_{\Omega_t}\int_{\Omega_r} E\left\{|g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})|^2\right\}d\hat{\boldsymbol{\phi}}d\hat{\boldsymbol{\varphi}}}. \qquad (12)$$

From (11), we can claim that correlation coefficient $E\{h_{r_1t}h^*_{r_2t}\}$ observed at the receiver satisfies the spatial correlation function given in (7) with normalization $\int_{\Omega_t}\int_{\Omega_r} E\left\{|g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})|^2\right\} d\hat{\boldsymbol{\phi}}d\hat{\boldsymbol{\varphi}} = 1$.

Correlation coefficients observed at the transmitter

A modelling approach similar to above can be employed to derive the correlation coefficients observed at the transmitter antenna elements. Note that the form of the azimuth power distribution observed at the transmitter could be different from that observed at the receiver.

Antenna correlation coefficients observed at the transmitter is given by

$$E\{h_{rt_1}h^*_{rt_2}\} = \int_{\Omega_t} \mathcal{P}_{\mathrm{Tx}}(\hat{\boldsymbol{\phi}})e^{-ik(\boldsymbol{x_{t_2}}-\boldsymbol{x_{t_1}}).\hat{\boldsymbol{\phi}}}d\hat{\boldsymbol{\phi}}, \qquad (13)$$

where $\mathcal{P}_{\mathrm{Tx}}(\hat{\boldsymbol{\phi}})$ is the normalized average power transmitted in to the direction $\hat{\boldsymbol{\phi}}$ with normalization $\int_{\Omega_t}\int_{\Omega_r} E\left\{|g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})|^2\right\}d\hat{\boldsymbol{\phi}}d\hat{\boldsymbol{\varphi}} = 1$ and

$$\mathcal{P}_{\mathrm{Tx}}(\hat{\boldsymbol{\phi}}) = \frac{\int_{\Omega_r} E\left\{|g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})|^2\right\}d\hat{\boldsymbol{\varphi}}}{\int_{\Omega_t}\int_{\Omega_r} E\left\{|g(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\varphi}})|^2\right\}d\hat{\boldsymbol{\phi}}d\hat{\boldsymbol{\varphi}}}. \qquad (14)$$

Thus (11) and (13) will help us to express the spatial correlation coefficients observed at the receiver/transmitter as a function of antenna separation and azimuth power distribution. In the next section we derive an approximation to these two equations in 2-D[2] space.

2-Dimensional Scattering environment

Consider the Jacobi-Anger expansion [7, page 67] for plane wave $e^{ik\boldsymbol{y}.\hat{\boldsymbol{\phi}}}$ given by

$$e^{ik\boldsymbol{y}.\hat{\boldsymbol{\phi}}} = \sum_{m=-\infty}^{\infty}[J_m(ky)e^{-im(\phi_y-\pi/2)}]e^{im\phi}, \qquad (15)$$

where $J_m(x)$ is the integer order m Bessel function, $\boldsymbol{y} \equiv (y,\phi_y)$ and $\hat{\boldsymbol{\phi}} \equiv (1,\phi)$ in polar coordinate system. Substitution of (15) into (7) gives the 2-D spatial correlation function

$$\rho(\boldsymbol{y_2}-\boldsymbol{y_1}) = \sum_{m=-\infty}^{\infty}\alpha_m J_m(k \parallel \boldsymbol{y_2}-\boldsymbol{y_1} \parallel)e^{im\phi_{12}}, \qquad (16)$$

[2] The 2-D case is a special case of 3-D case where all the signals arrive from horizontal plane only.

where ϕ_{12} is the angle between the vectors connecting $\boldsymbol{y_1}$ and $\boldsymbol{y_2}$. The Fourier coefficients α_m characterize the 2-D scattering environment surrounding the antenna array and are given by

$$\alpha_m = i^m\int_0^{2\pi}\mathcal{P}(\phi)e^{-im\phi}d\phi, \qquad (17)$$

where $\mathcal{P}(\phi)$ is the normalized angular power distribution or the azimuth power distribution given in (8). It was shown in [8] that to compute the spatial correlation $\rho(\boldsymbol{y_2}-\boldsymbol{y_1})$, only $2M+1$ terms in the sum (16) need to be evaluated and the truncated series gives the best approximation to the spatial correlation. The truncation point M is given by $\lceil \pi ed/\lambda \rceil$, where $\lceil . \rceil$ is the ceiling operator, d is the separation between two antennas and $e \approx 2.7183$. Then the spatial correlation function is

$$\rho(\boldsymbol{y_2}-\boldsymbol{y_1}) = \sum_{m=-M}^{M}\alpha_m J_m(k \parallel \boldsymbol{y_2}-\boldsymbol{y_1} \parallel)e^{im\phi_{12}}. \qquad (18)$$

Non-Isotropic Scattering Environment

One of the most commonly used distributions is the isotropic scattering model, where the power is assumed to be arriving uniformly from all angles of arrival (AOA) [9]. However the existence of such a model is highly unlikely due to the nature of the surrounding scattering environments. Several azimuthal power distributions have been proposed in literature for modelling the non-isotropic scattering in 2-D space. For example, Uniform-Limited, Gaussian, von-Mises, Laplacian, Cylindric Harmonic [6]. It was shown in [10] that all power distribution models give very similar spatial correlation for a given angular spread. Therefore, we restrict our performance investigation of the O-STBC only to Uniform-limited power distribution case, where the energy is arriving from restricted range of azimuth angles $\pm\triangle$ around a mean AOA $\psi_0 \in [-\pi,\pi)$. The power density is given by $\mathcal{P}(\psi) = \frac{1}{2\triangle}$, $|\psi-\psi_0| \leq \triangle$. The angular spread, σ, is the standard deviation of the distribution, which is related to the non-isotropy factor of the distribution. For the Uniform-limited distribution, $\sigma^2 = \frac{1}{3}\triangle^2$. The scattering environment coefficients α_m (17) are given by

$$\alpha_m = \frac{\sin(m\triangle)}{m\triangle}e^{im(\pi/2-\psi_0)}. \qquad (19)$$

Note that, isotropic factor $\triangle = \pi$ or angular spread $\sigma = \pi/\sqrt{3}$ ($\approx 104°$) represents the isotropic scattering environment, in other words, the 2-D omnni-Directional diffuse field, where the spatial correlation between any two antenna elements is given by $\rho(\boldsymbol{y_2}-\boldsymbol{y_1}) = J_0(k \parallel \boldsymbol{y_2}-\boldsymbol{y_1} \parallel)$ [9].

Correlated channel gains for a 2×2 MIMO system

We investigate the performance of O-STBC for two transmitter antennas and two receiver antennas. The restriction to two transmitter antennas allows us to use the complex-orthogonal block code proposed by Alamouti[3] [1].

Consider a 2×2 MIMO system with correlated channel gain matrix $\mathbf{H}$, where entries of $\mathbf{H}$ are given by (1). Normalized

[3] For the Alamouti O-STBC scheme, there is no restriction to the number of antennas at the receiver, but for simplicity we restrict to two receiver antennas.

spatial correlation matrices observed at the transmitter and the receiver antenna elements are

$$\mathbf{R}_{\mathrm{Rx}} = \begin{pmatrix} 1 & \rho \\ \rho^* & 1 \end{pmatrix}, \qquad \mathbf{R}_{\mathrm{Tx}} = \begin{pmatrix} 1 & \mu \\ \mu^* & 1 \end{pmatrix},$$

where $\rho = E\{h_{11}h_{21}^*\} = E\{h_{12}h_{22}^*\}$ and $\mu = E\{h_{11}h_{12}^*\} = E\{h_{21}h_{22}^*\}$. Then the correlation matrix $\mathbf{R}_{\mathrm{H}}$ of the MIMO channel $\mathbf{H}$ is found from (5), where $\mathbf{R}_{\mathrm{H}} = \mathbf{R}_{\mathrm{Tx}} \otimes \mathbf{R}_{\mathrm{Rx}}$. Since $\mathbf{R}_{\mathrm{H}}$ is Hermitian, it can be Cholesky factorized to obtained the lower triangular matrix $\mathbf{C}$ such that $\mathbf{R}_{\mathrm{H}} = \mathbf{C}\mathbf{C}^T$. Let $\mathbf{A}$ be 2×2 uncorrelated MIMO channel matrix and $\boldsymbol{a} = \mathrm{vec}(\mathbf{A}) = [a_{11}, a_{12}, a_{21}, a_{22}]$, where elements in $\mathbf{A}$ are i.i.d complex zero-mean gaussian variables with unity variance. Using the method described in section III, we may derive the correlated channel matrix $\mathbf{H}$ as

$$\mathbf{H} = \begin{pmatrix} a_{11} & \rho^* a_{11} + \beta a_{12} \\ \mu^* a_{11} + \gamma a_{21} & \mu^*\rho^* a_{11} + \mu^*\beta a_{12} + \rho^*\gamma a_{21} + \beta\gamma a_{22} \end{pmatrix},$$

where $\beta = \sqrt{1 - |\rho|^2}$, and $\gamma = \sqrt{1 - |\mu|^2}$. By taking the covariance between channel gains h_{ij} in $\mathbf{H}$, we can easily show that the correlated channel gains satisfy $\mathbf{R}_{\mathrm{H}}$. Here ρ and μ are the spatial correlation coefficients defined in (11) and (13), respectively.

For reasons of simplicity, we confine our investigations to the case where zero spatial correlation observed at the transmitter antenna elements (i.e., $\mathbf{R}_{\mathrm{Tx}} = I_{2\times 2}$) and arbitrary spatial correlation observed at the receiver antenna elements.

IV. Simulation Results

We first investigate the effects of antenna separation on O-STBC. We compare the bit-error rate performance of a 2×2 orthogonal space-time block coded system against a 1×2 un-coded system. On the un-coded system, the Maximum Ratio Receive Combining (MRRC) together with maximum-likelihood detection, is employed at the receiver to detect receiving symbols. We set the overall SNR, before detection of each symbol, to 10dB and angular spreads to $\sigma = [104°, 20°, 5°]$ for a Uniform-limited distribution and increase the separation distance between receiver antennas, which are positioned on the x-axis. The spatial correlation coefficients are calculated using (18), where the angle ϕ_{12} is zero as we position all our antennas on the x-axis. The performance results for both coded and un-coded systems are shown in Fig. 2. For all angular spreads, the bit-error rate of the coded system varies from 0.002 to 0.007, giving an overall error range of 0.005, whereas the bit-error rate of the un-coded system varies from 0.007 to 0.0295, giving an overall error range of 0.0225. So the range of the bit-error rate given by the un-coded system is quite significant, compared to that of orthogonal coded system, with the increase in antenna separation. Indeed the orthogonal coded system's overall error range is not considerably significant, which means the antenna separation plays a secondary role in the performance of O-STBC. In other words, most of the error rate improvement of O-STBC is due to the time-coding rather than to the space-coding. As shown, the orthogonal coded system reaches its optimum performance, 0.002, when the antenna separation distances are λ, 1.5λ and 3λ for angular spreads 104°, 20° and 5° respectively. Hence we can claim that the role of the space in O-STBC is limited, even though the antenna separation is a main contributor to the spatial correlation between antenna elements. This corroborates the claim that the O-STBC has the maximum resistance against the spatially correlated fading.

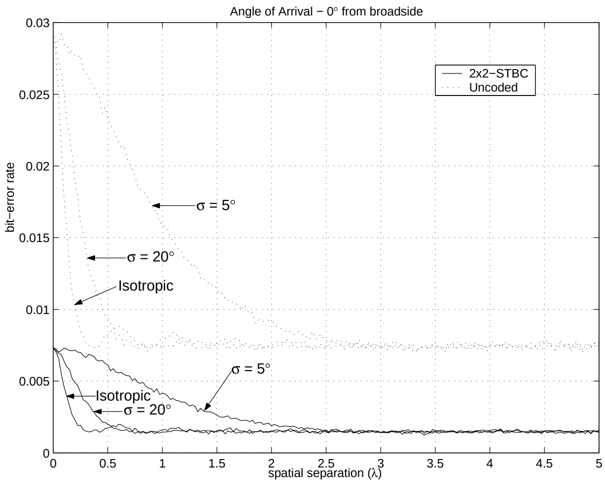

Fig. 2: Bit-error rate vs spatial separation for 2×2 O-STBC and 1×2 un-coded SIMO system for a Uniform-limited distribution. Mean AOA $\psi_0 = 0°$ from broadside, angular spread $\sigma = \{104°, 20°, 5°\}$ and SNR = 10dB.

It is observed that coded system reaches the optimum performance of un-coded system when the antenna separation is zero. Zero antenna separation will result in full spatial correlation at the receiver antennas. O-STBC scheme assumes that the receiver has the perfect knowledge of the channel. In practise, receiver estimates the channel using periodic training symbols transmitted by transmitter antennas. However when the receiver is in full spatial correlation, it may not be possible to estimate the channel as the received signal energy could be zeroed out by the channel. This is a potential draw back to the O-STBC scheme which could impact heavily on the performance. However, this is equally true for the un-coded system as well.

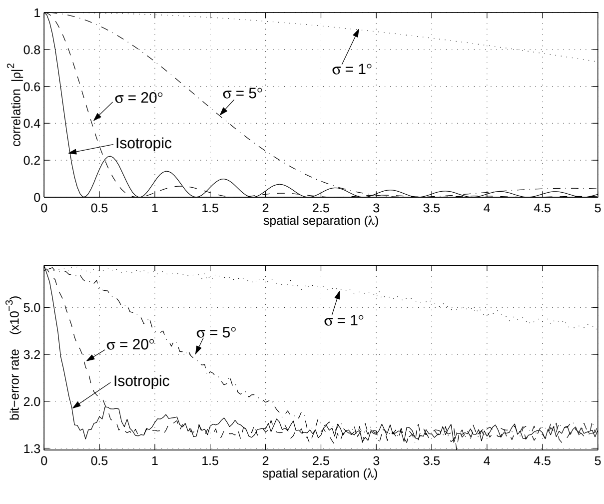

Fig. 3: **(a)**. Spatial correlation between two receiver antennas positioned on the x-axis for mean AOA $\psi_0 = 0°$ from broadside vs the spatial separation for a uniform-limited scattering distribution with angular spreads $\sigma = [104°, 20°, 5°, 1°]$. **(b)**. Bit-error rate vs spatial separation for 2×2 O-STBC under the scattering environment in (a)

The performance results for 2×2 Alamouti code for mean AOA $\psi_0 = 0°$ (broadside) is shown in Fig. 3. Here we

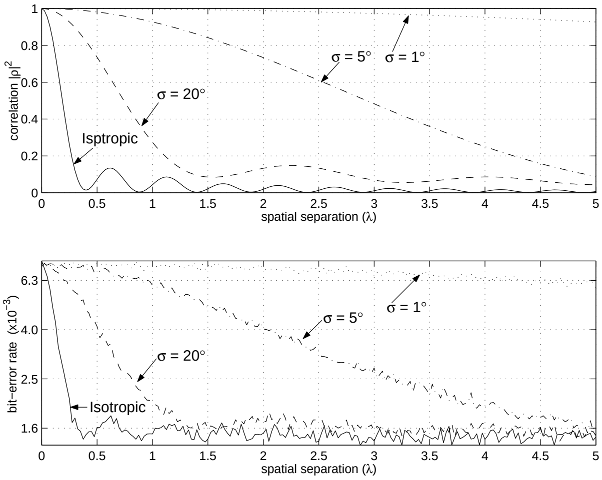

Fig. 4: **(a)**. Spatial correlation between two receiver antennas positioned on the x-axis for mean AOA $\psi_0 = 60^\circ$ from broadside against the spatial separation for a uniform-limited scattering distribution with angular spreads $\sigma = [104^\circ, 20^\circ, 5^\circ, 1^\circ]$. **(b)**. Bit-error rate vs spatial separation for 2×2 O-STBC under the scattering environment in (a)

set the overall SNR to 10dB and angular spread to $\sigma = [104^\circ, 20^\circ, 5^\circ, 1^\circ]$ for a Uniform-limited distribution where antennas are positioned on the x-axis. As shown, the bit-error rate decreases as the antenna spacing and the angular spread, σ, increase. It is observed that the bit-error rate performance does not decrease monotonically with antenna separation, for example, when $\sigma = 104^\circ$ (isotropic distribution) and 20°. The performance of the O-STBC is lower when the angular spread is smaller. This is due to the higher concentration of energy closer to the mean AOA for smaller angular spreads. Thus the angular spread is a major contributor to the bit-error rate performance of the O-STBC. To achieve most of the performance gain from O-STBC for small angular spread like 5°, as a rule of thumb, antennas in an aperture must be located at least 3λ apart from each other. Finally, we observe that the bit-error rate performance is directly mapped to the squared spatial correlation against the spatial separation for all angular spreads. In other words, bit-error rate performance has a strong correlation with the squared spatial correlation.

Fig. 4 shows the performance results for mean AOA $\psi_0 = 60^\circ$ from broadside. Here we observe similar results as for the mean AOA $\psi_0 = 0^\circ$. In this case, a significant performance degradation is observed for all angular spreads for the same antenna separation as for previous results. So the performance of the O-STBC is decreased as the mean AOA moves away from the broadside. This can be justified by the reasoning that as the mean AOA moves away from the broadside, there will be a reduction in the angular spread exposed to antennas and hence less signals being captured. Under this environment, antennas must be placed at least 4.5λ apart from each other to achieve most of the performances gain provided by the O-STBC.

V. Conclusions

We showed that the spatial correlation coefficients between channel gains can be expressed as a function of antenna separation and surrounding azimuth power distribution. This facilitates realistic modelling in an analytic framework.

The performance gain achieved from O-STBC is not considerably significant with the increase in antenna separation. It corroborates the claim that O-STBC provides a high degree of robustness against the spatially correlated fading, even though the antenna separation is a major contributor to spatial correlation. In order to achieve the optimal performance given by the O-STBC, antennas in an aperture must be placed at least 4.5λ distance apart. This 'rule-of-thumb' caters for power distributions with narrow angular spreads like 5°. We showed that bit-error rate does not decrease monotonically with antenna separation. In fact, bit-error rate performance against the spatial separation appears to be one-to-one mapped with the squared spatial correlation against the spatial separation. The performance of O-STBC is decreased when the mean AOA of an impinging signal moves away from the broadside. Further the bit-error rate is higher when the angular spread of the azimuth power distribution is smaller.

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