Effect of Transmit Antenna Configuration on Rank-Determinant Criteria of Space-time Trellis Codes

Tharaka A. Lamahewa, Thushara D. Abhayapala and Rodney A. Kennedy

***Abstract*— In this paper we derive a new upper bound for the pairwise error probability of space-time codes in a quasi-static Rayleigh fading channel, considering antenna configuration at the transmitter side. The design criterion for existing space-time trellis codes is based on the rank and the determinant of the distance matrix between two code words. In particular, the diversity advantage of the space-time code is associated to the rank of the distance matrix. We show that when the transmit antenna region is small, the diversity advantage given by the space-time code is reduced by the transmit antenna configuration and the diversity advantage of the code depends on the rank of the antenna configuration matrix. We also show that the uniform linear array antenna configuration diminishes the diversity advantage provide by the space-time code while the uniform circular array antenna configuration does not effect to the diversity advantage of the space-time code.**

***Index Terms*— Space-time trellis codes, antenna configuration, pairwise error probability.**

I. Introduction

Pairwise error probability (PEP) of space-time codes in quasi-static Rayleigh fading channels is determined by two parameters, which are the rank and the determinant of the distance matrix between two code words [1]. Space-time trellis codes are designed based on the rank determinant criteria which involves maximizing the minimum rank and the minimum determinant of the distance matrix over all distinct pairs of code words. Based on the above design criteria, Tarokh *et al.* [1] proposed several hand-designed QPSK and 8PSK space-time trellis codes for transmission using two transmit antennas in independent flat fading channels. In [2], space-time trellis codes for more than two transmit antennas are proposed based on the same design criteria.

When deriving space-time trellis codes, one of the main assumptions being made is, channel gains between the transmitter and receiver antennas undergo independent flat fading. Such an assumption is valid only if the scattering environment is isotropic, i.e. scattering is uniformly distributed over the receiver and transmitter antenna arrays and also the antennas in an array are separated at a distance greater than $\lambda/2$, where λ is the wave length [3]. However, if we consider a multi-antenna Mobile Unit (MU), then the antennas at the MU cannot be spaced sufficiently well apart due to the limited size of the MU. This has motivated us to investigated the effect of antenna separation as well as the antenna configuration (e.g., Uniform Linear, Uniform Circular, Uniform Grid, etc) on the performance of space-time trellis codes.

In this paper, we derive a new upper bound for the PEP of space-time coded system with n_T-transmit antennas and one-receive antenna, using a realistic channel model which fully accounts for antenna separation, antenna configuration and scattering environment. Traditionally the design criterion for existing space-time trellis codes is based on only the rank and the determinant of the distance matrix between two code words. Here we show that the transmitter antenna configuration should also be taken account in the rank determinant design criteria of space-time trellis codes when fading is not independent. We evaluate the frame error rate performance of space-time trellis codes proposed in [2] for different antenna configurations. We show that the uniform linear array (ULA) antenna configuration diminishes the diversity gain provided by the space-time trellis codes while the uniform circular array (UCA) antenna configurations retain the diversity gain provided by the space-time trellis codes when more than two transmit antennas being used for transmission.

II. System Model

We consider a base-band mobile communication system which employs n_T transmit antennas and one-receive antenna. The data transmitted from n_T transmit antennas are encoded by a space-time trellis code. Assume that at each time t, space-time encoder produces n_T outputs $\boldsymbol{x}_t = [x_{1,t}, x_{2,t}, \ldots, x_{n_T,t}]^T$, where $x_{i,t}$ is a signal from a certain constellation with unit energy. These outputs are then simultaneously transmitted from n_T transmit antennas. Assuming quasi-static fading, the signal received at the receiver during L symbol periods can be expressed in matrix form as

$$\boldsymbol{r} = \sqrt{E_s}\boldsymbol{h}\mathbf{X} + \boldsymbol{z}, \tag{1}$$

where E_s is the transmitted power per symbol at each transmit antenna and $\boldsymbol{h}$ is the $1\times n_T$ transfer matrix of the channel with entries h_i, where h_i is the fading coefficient between transmit antenna i and the receive antenna, $\boldsymbol{r} = [r_1, r_2, \ldots, r_L]$ and $\boldsymbol{z} = [z_1, z_2, \ldots, z_L]$ are $1\times L$ matrices and $\mathbf{X}$ is the $n_T\times L$

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transmitted code word, which has the form

$$\mathbf{X} = \begin{pmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,L} \\ x_{2,1} & x_{2,2} & \dots & x_{2,L} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n_T,1} & x_{n_T,2} & \dots & x_{n_T,L} \end{pmatrix}, \quad (2)$$

where $\boldsymbol{x}_{m,l}$ is the complex valued modulation symbol[1] transmitted from antenna m at symbol interval l. We assume that $\boldsymbol{z}$ is zero-mean Additive White Gaussian Noise with covariance matrix $\mathbf{R}_{\boldsymbol{z}} = E\left\{\boldsymbol{z}^H\boldsymbol{z}\right\} = N_0\mathbf{I}$, where $\mathbf{I}$ is the identity matrix and $E\{\cdot\}$ is the mathematical expectation.

In this paper, we use the 2-dimensional spatial channel model proposed in [4] to explore the spacial aspects of space-time trellis codes. In this spatial channel model, MIMO channel is separated in to three physical regions of interest: scatterer free region around the transmitter antenna array, scatterer free region around the receiver antenna array and the complex random scattering media which is the complement of the unions of two antenna array regions. In other words, MIMO channel is decomposed into deterministic and random matrices, where the deterministic portion depends on the physical configuration of the transmitter and the receiver antenna arrays and the random portion represents the complex scattering media between the transmitter and the receiver antenna regions. In our analysis, we use a system which employs n_T-transmit antennas and one-receive antenna. Assume that the receiver antenna is positioned at the center of the receiver antenna region. Then the spatial channel model in [4] reduces to

$$\boldsymbol{h} = \boldsymbol{h}_s\mathbf{J}_T^{\dagger}. \quad (3)$$

where $\mathbf{J}_T$ is the transmitter configuration matrix (or the transmit aperture sampling matrix) which includes antenna positions and antenna orientation relative to the transmitter aperture origin, $[\cdot]^{\dagger}$ denotes the matrix conjugate transpose and $\boldsymbol{h}_S$ is the complex scattering gain matrix (in this case a vector).

The transmitter antenna configuration matrix, $\mathbf{J}_T$, is

$$\mathbf{J}_T = \begin{pmatrix} \mathcal{J}_{-M_T}(\boldsymbol{x}_1) & \dots & \mathcal{J}_{M_T}(\boldsymbol{x}_1) \\ \mathcal{J}_{-M_T}(\boldsymbol{x}_2) & \dots & \mathcal{J}_{M_T}(\boldsymbol{x}_2) \\ \vdots & \ddots & \vdots \\ \mathcal{J}_{-M_T}(\boldsymbol{x}_{n_T}) & \dots & \mathcal{J}_{M_T}(\boldsymbol{x}_{n_T}) \end{pmatrix}, \quad (4)$$

where $\mathcal{J}_n(\boldsymbol{x})$ is the spatial-to-mode[2] function (SMF), which is related to the shape of the scatterer free antenna region. For a circular region in 2D space, the SMF is given by a bessel function of first kind [4] and for a sphere region in 3D space, the SMF is given by a spherical bessel function [5]. For a prism shape region, the SMF is given by a prolate spheroidal function [6].

[1] Modulation symbols are taken from a signal constellation with unit energy.

[2] The set of modes forms a basis of functions for representing a multipath wave field.

In this paper, we consider the 2-dimensional[3] space, where all the transmit antennas are encompassed in a circular aperture with a finite radius. Then the spatial-to-mode function for the circular aperture is given by

$$\mathcal{J}_n(\boldsymbol{x}) \triangleq J_n(k\|\boldsymbol{x}\|)e^{in(\phi_x - \pi/2)},$$

where $J_n(\cdot)$ is the Bessel function of integer order n, vector $\boldsymbol{x} = (\|\boldsymbol{x}\|, \phi_x)$, in polar coordinates is the antenna location relative to the origin of the aperture which encloses the transmit antennas, $k = 2\pi/\lambda$ is the wave number with λ being the wave length and $i = \sqrt{-1}$. Note that $\mathbf{J}_T$ is a $n_T \times (2M_T+1)$ matrix, where n_T is the number of transmit antennas and $(2M_T+1)$ is the number of effective communication modes at the circular transmit aperture. M_T is given by [7]

$$M_T \triangleq \lceil \pi e r_T/\lambda \rceil, \quad (5)$$

where r_T is the minimum radius of the circular aperture which encompass all transmit antennas. Note that $\mathbf{J}_T$ is fixed and known for a given transmit antenna configuration. Also note that number of effective communication modes at the transmitter region is determined by the size of the region but not from the number of antennas available for transmission [7].

The m-th element of $1\times(2M_T+1)$ scattering gain vector $\boldsymbol{h}_S$ is given by

$$\gamma_m = \{\boldsymbol{h}_S\}_m = \int_{\mathbb{S}^1} g(\phi)e^{i(m-M_T-1)\phi}d\phi$$
$$m = 1, 2, ..., (2M_T+1),$$

where $g(\phi)$ is the effective random complex scattering gain function for a signal leaving the transmitter aperture at an angle ϕ. Note that, for a rich scattering environment $\{\gamma_m\}$ can be assumed to be independent each other and can be modelled as complex Gaussian random variables. Note that equation (3) plays an integral part of this paper.

In the next section we derive a new upper bound for the PEP of space-time codes for the channel in (3).

III. A New Upper Bound for the Pairwise Error Probability

In this section, we derive a new upper bound for the PEP of a space-time coded system with n_T-transmit antennas and one-receive antenna. The approach we take to develop this new upper bound is similar to that presented in [1]. Assume that perfect channel state information (CSI) is available at the receiver and a maximum likelihood (ML) receiver is employed. Assume the codeword $\mathbf{X}$ in (2) was transmitted, but the ML-decoder chooses another codeword $\hat{\mathbf{X}}$, then the PEP conditioned on the channel $\boldsymbol{h}$, can be written as

$$\begin{aligned} \mathrm{P}(\mathbf{X} \to \hat{\mathbf{X}}|\boldsymbol{h}) &= \mathrm{P}(\|\boldsymbol{r} - \sqrt{E_s}\boldsymbol{h}\hat{\mathbf{X}}\|^2 < \|\boldsymbol{r} - \sqrt{E_s}\boldsymbol{h}\mathbf{X}\|^2), \\ &= \mathrm{P}(\mathrm{Re}\{\boldsymbol{h}(\hat{\mathbf{X}} - \mathbf{X})\boldsymbol{z}^{\dagger}\} > \sqrt{\frac{E_s}{4}}d^2(\mathbf{X}, \hat{\mathbf{X}})), \end{aligned}$$

[3] The 2-dimensional case is a special case of 3-dimensional case where all the signals arrive from or depart to horizontal plane only.

where $d^2(\mathbf{X}, \hat{\mathbf{X}}) = \boldsymbol{h}(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^{\dagger} \boldsymbol{h}^{\dagger}$, $\mathrm{Re}\{\cdot\}$ is the real part of an argument and $\|\cdot\|$ is the Eucleadian norm. Then the PEP can be upper bounded by the *Chernoff bound* [8, page 127]

$$\mathrm{P}(\mathbf{X} \to \hat{\mathbf{X}}|\boldsymbol{h}) \leq \exp\left(-\frac{E_s}{4N_0} d^2(\mathbf{X}, \hat{\mathbf{X}})\right). \tag{6}$$

Using the spatial channel model, $\boldsymbol{h} = \boldsymbol{h}_s \mathbf{J}_T^{\dagger}$, we can write $d^2(\mathbf{X}, \hat{\mathbf{X}}) = \boldsymbol{h}_s \mathbf{A} \boldsymbol{h}_s{}^{\dagger}$, where

$$\mathbf{A} = \mathbf{J}_T^{\dagger}(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^{\dagger} \mathbf{J}_T. \tag{7}$$

Since $\mathbf{A}$ is a Hermitian matrix (i.e. $\mathbf{A} = \mathbf{A}^{\dagger}$), there exist a unitary matrix $\mathbf{V}$ and a real diagonal matrix $\mathbf{D}$ such that $d^2(\mathbf{X}, \hat{\mathbf{X}}) = \boldsymbol{h}_s \mathbf{V}^{\dagger} \mathbf{D} \mathbf{V} \boldsymbol{h}_s^{\dagger}$. Diagonal entries of $\mathbf{D}$ are the distinct eigenvalues of $\mathbf{A}$, i.e., $\lambda_m,\ m = 1, 2, \ldots, (2M_T + 1)$. Let $\boldsymbol{\beta} = \boldsymbol{h}_s \mathbf{V} = [\beta_1, \beta_2, \ldots, \beta_{2M_T+1}]$, then (6) can be written as

$$\mathrm{P}(\mathbf{X} \to \hat{\mathbf{X}}|\boldsymbol{h}) \leq \exp\left(-\frac{E_s}{4N_0} \sum_{m=1}^{2M_T+1} \lambda_m |\beta_m|^2\right). \tag{8}$$

For a rich scattering environment, we can model elements of $\boldsymbol{h}_s$ as zero-mean independent identically distributed complex Gaussian random variables each with unit variance. Since $\mathbf{V}$ is unitary, $\{|\beta_m|\}$ are also zero-mean independent complex Gaussian random variables each with variance one. Then $|\beta_m|$ are distributed according to a Rayleigh distribution with pdf

$$f(|\beta_m|) = 2|\beta_m| \exp(-|\beta_m|^2), \quad \text{for} \quad |\beta_m| \geq 0.$$

The to compute the upper bound, we average (8) with respect to independent Rayleigh distributions of $|\beta_m|$ to arrive at

$$\mathrm{P}(\mathbf{X} \to \hat{\mathbf{X}}) \leq \prod_{m=1}^{2M_T+1} \left(1 + \frac{E_s}{4N_0} \lambda_m\right)^{-1}. \tag{9}$$

Let q denote the rank of matrix $\mathbf{A}$ in (7). Then matrix $\mathbf{A}$ has q non-zero eigenvalues including multiplicity. It follows from the inequality (9) that for high SNR, we obtain the upper bound of the PEP

$$\begin{aligned} \mathrm{P}(\mathbf{X} \to \hat{\mathbf{X}}) &\leq \left(\frac{E_s}{4N_0}\right)^{-q} \left(\prod_{m=1}^{q} \lambda_m\right)^{-1} \\ &= \left(\frac{E_s}{4N_0}\right)^{-q} \frac{1}{\det[\mathbf{J}_T^{\dagger}(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^{\dagger} \mathbf{J}_T]}. \end{aligned} \tag{10}$$

Equation (10) suggests that, we can achieve a diversity advantage of q and a coding advantage of $(\lambda_1 \lambda_2 \ldots \lambda_q)^{1/q}$. So the rank of matrix $\mathbf{A}$ plays a major role in determining the diversity advantage as well as the coding advantage of a space-time coded system. For the matrix $\mathbf{A}$, we have

$$\mathrm{rank}\{\mathbf{A}\} = \min\{\mathrm{rank}\{\mathbf{J}_T\};\ \mathrm{rank}\{(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^{\dagger}\}\} \tag{11}$$

Most of the space-time trellis codes derived so far were based on maximizing the minimum rank and the minimum product of all the none zero eigenvalues of matrix

$$\mathbf{Y} = (\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^{\dagger}. \tag{12}$$

Most of the codes presented in [1], [2], [9] achieve the full rank of $\mathbf{Y}$, which is the number of transmit antennas.

We now illustrate the effect of antenna configuration on the performance of space-time trellis codes. Consider a space-time trellis code derived based on the rank determinant criteria given in [1]. Assume that this code gives a diversity advantage of r (for one receive antenna), which is the rank of matrix $\mathbf{Y}$. Also assume n_T transmit antennas are placed in some configuration and the rank of the transmit antenna configuration matrix, $\mathbf{J}_T$, is $q(< r)$. According to the upper bound (10), the diversity advantage of the system is determined by the rank of $\mathbf{A} = \mathbf{J}_T^{\dagger}(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^{\dagger} \mathbf{J}_T$. Then from (11), we can observe that the rank of matrix $\mathbf{A}$ is q. So the rank reduction of the code due to the antenna configuration will result in lower diversity advantage than the diversity advantage given by the space-time code itself.

This raises two questions.

- Can we have a transmit antenna configuration matrix whose rank is less than the rank of $\mathbf{Y}$ (or number of transmit antennas in the case of full rank $\mathbf{Y}$)?
- For a given region, what is the best antenna configuration type that is suited to employ a space-time code, where the antenna configuration does not reduce the diversity advantage given by the space-time code?

If we have less number of effective communication modes available at the transmitter aperture than the number of transmit antennas, then the rank of $\mathbf{J}_T$ will be less than or equal to the number of modes. From (5), it is obvious that the number of modes available at the transmitter aperture is solely dependent on the size of the transmitter antenna region (or radius of the circular aperture) but not on the number of transmit antennas being used. This implies that the lesser the radius of the region, the lesser the number of available effective modes at the transmit aperture. Therefore, the number of modes available at the transmitter aperture indirectly limits the performance of space-time trellis codes by reducing the overall rank of $\mathbf{A}$.

In the next section we provide simulation results for space-time trellis codes with three and four transmit antennas to support the argument we developed above.

IV. Simulation Results

We now present the performance results for space-time trellis codes with three and four transmit antennas to support the theoretical work of the previous section. Performance is measured in terms of frame error rates. Note that our objective is to investigate the performance of space-time trellis codes when the size of the transmit aperture is quiet small (for e.g., antennas on a mobile unit). Therefore in our simulations we consider circular regions with radius of 0.1λ and 0.2λ. We consider the following two space-time trellis codes presented in [2, Table 1].

(a) 4-PSK, 16-states trellis code with three transmit antennas. $\mathrm{rank}(\mathbf{Y}) = 3$ and $\det(\mathbf{Y}) = 32$.
(b) 4-PSK, 64-states trellis code with four transmit antennas. $\mathrm{rank}(\mathbf{Y}) = 4$ and $\det(\mathbf{Y}) = 64$.

With a single receiver antenna, diversity advantage obtained from code-(a) is 3 and code-(b) is 4.

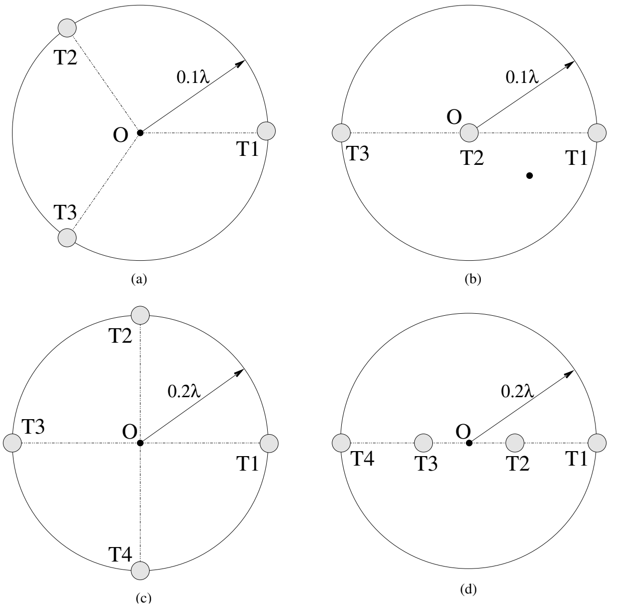

Fig. 1. (a)/(c)-Three/ Four transmit antennas in an uniform circular array with radius of 0.1λ/ 0.2λ (b)/(d)-Three/ Four transmit antennas in an uniform linear array with radius of 0.1λ/ 0.2λ.

For code-(a), we place the three transmit antennas in a UCA and also in a ULA as depicted in Fig-1(a) and Fig-1(b) respectively. For both configurations, we set the radius of the circular aperture to 0.1λ, corresponding to $2\lceil \pi e 0.1\rceil + 1 = 3$ effective modes at the transmit aperture. We found that $\mathrm{rank}(\mathbf{J}_T) = 3 = \mathrm{rank}(\mathbf{Y})$ for UCA antenna configuration and $\mathrm{rank}(\mathbf{J}_T) = 2(< \mathrm{rank}(\mathbf{Y}))$ for ULA antenna configuration. The performance results of code-(a) for these two antenna configurations are shown in Fig-2. On the same figure, we also show the performance results of code-(a) without considering the antenna configuration. In fact, here we assume that antennas are placed far apart and the correlation between antenna elements due to space is zero. From Fig-2, we can observe that the rank of the antenna configuration matrix $\mathbf{J}_T$ effects the performance of space-time trellis codes. Furthermore, we observe that as the SNR increases, the performance difference between two configurations are quiet significant.

Fig-2 also suggests that at 0.1λ radius with three transmit antennas, the UCA antenna configuration is best suited to employ space-time trellis codes, as it does not diminish the diversity gain provided by the code, where as the ULA configuration is not suited as it reduces the diversity advantage given by the space-time trellis code since the rank of $\mathbf{J}_T$ is less than the rank of $\mathbf{Y}$. We can also observe that there is a significant performance difference between the ideal case and the UCA antenna configuration. The reason for this difference is that, in the ideal case we assume transmit antennas are located far apart from each other, while in the UCA case all the transmit antennas are enclosed in a circular region having a radius of 0.1λ. This will result in spatial correlation among transmit antenna elements and hence limiting the performance of the system. We also observed that, as we increase the radius of the transmit circular aperture, the number of effective communication modes at the transmit aperture increase. As a result of this increment, the rank of $\mathbf{J}_T$ becomes equal to the number of transmit antennas, for both antenna configurations, which gives no impact from transmit antenna configuration to the diversity advantage given by the code. However the performance of the code is still limited due to the finite antenna separation.

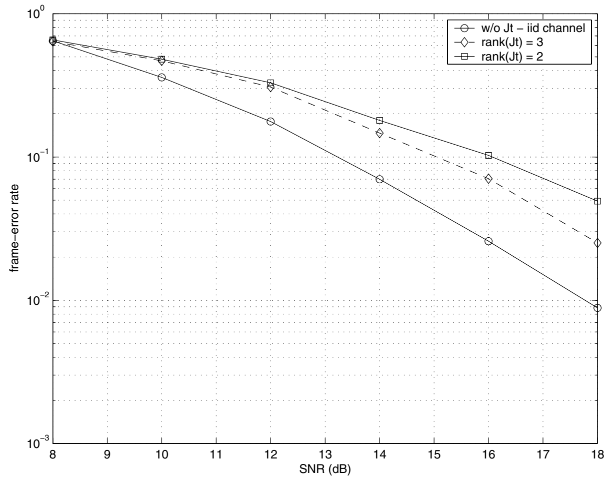

Fig. 2. Frame error rate performance of the 4-PSK, 16 states space-time trellis code with three transmit antennas for UCL, ULA antenna configurations and ideal channel case.

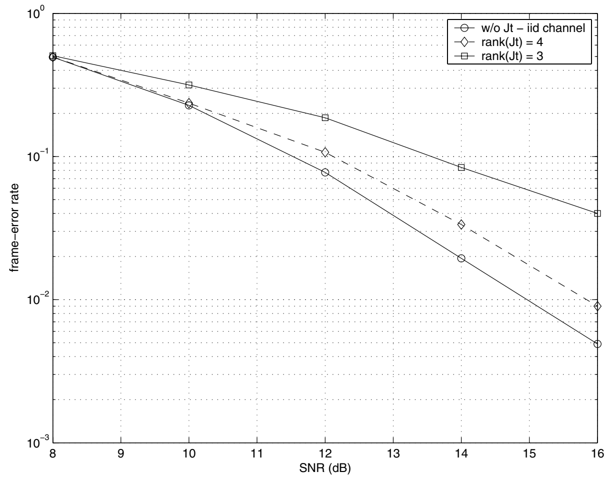

Fig. 3. Frame error rate performance of the 4-PSK, 64 states space-time trellis code with four transmit antennas for UCL, ULA antenna configurations and ideal channel case.

For code-(b), we place the four transmit antennas as depicted in Fig-1(c) and Fig-1(d). For both configurations we set the radius of the circular aperture to 0.2λ, corresponding to $2\lceil \pi e 0.2\rceil + 1 = 5$ effective modes at the transmit aperture. It is found that $\mathrm{rank}(\mathbf{J}_T) = 3(< \mathrm{rank}(\mathbf{Y}))$ for the ULA antenna configuration and $\mathrm{rank}(\mathbf{J}_T) = 4 = \mathrm{rank}(\mathbf{Y})$ for the UCA antenna configuration. The performance results of code-(b) for these two antenna configurations are shown in Fig-3. Similar performance results are observed as for the code-(a). We observe that at 0.2λ radius with four transmit antennas, UCA antenna configuration is best suited to employ space-time trellis codes while ULA antenna configuration is not.

We found that space-time trellis codes with two transmit antennas does not suffer from diversity loss due to the transmit antenna configuration. The reason for this is, matrix $\mathbf{J}_T$ is always rank two regardless of the antenna configuration.

V. Conclusions

A new upper bound for the pairwise error probability of a space-time code is derived for a system with multiple transmit antennas and one-receive antenna, considering antenna configuration at the transmitter. We showed that the rank of the antenna configuration matrix effects the diversity advantage given by the space-time trellis codes when the transmit antenna region is small. We also showed that, uniform circular antenna configuration is best suited to employ space-time trellis codes with more than two transmit antennas, as it does not reduce the diversity gain provided by the code. However we found that, the uniform linear array antenna configuration reduces the diversity advantage given by the code, resulting lower performance than the uniform circular antenna configuration.

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