Proceedings of RAAD'03, 12th International Workshop on Robotics in Alpe-Adria-Danube Region
Cassino May 7-10, 2003

EQUIVALENT KINEMATIC CHAINS WITH PLANAR-SPHERICAL BONDS APPLICATION TO THE DEVELOPMENT OF A 3-DOF 3-*RPS* PARALLEL MECHANISM

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ABSTRACT- The paper aims to analyze the equivalent kinematic chains of a family of three-degree-of-freedom (3-DOF) tripod mechanisms with planar-spherical bonds in order to determine the platform motions generated by the mechanisms. After a short introduction to mechanical generators of Lie subgroups of displacement, the mobility formula of a general 3-DOF tripod mechanism based on the modified Grüebler's criterion is given. Using displacement group theory theorems, the analyzed closed-loop system becomes finally equivalent to a 3 sphere-plane contacts. This will enable further design of new parallel mechanisms based on the task requirements. A prototype of a 3-DOF 3-*RPS* type parallel mechanism is designed and fabricated based on the equivalent kinematic chain analysis and the simulation result of workspace volume computation.

Keywords: Parallel Mechanism, Group Theory, Mobility, Mechanical Bond, Kinematic Design.

1 INTRODUCTION

During the past few years, there has been an increasing demand in the field of precision engineering for fine motion of multi-degrees of freedom. This motivates the development of a new robotics application field, parallel mechanism in the Mechanical Engineering Division at NgeeAnn Polytechnic, Singapore. The choice of parallel structures for high precision applications is justified by numerous advantages:

- High stiffness and structural frequency.
- Precision.
- Mobility and compactness.
- Fixed actuators.
- Uniform distribution of the load.

However, the main disadvantage is a limited working space. The simplest way to cumulate precision and working volume consists in the utilization of a parallel manipulator combined with classic-serial robots, as the active wrist [1]. The parallel manipulator thus compensates the static errors of the sequential robot with serial structure. This principle is often described by the name "Macro-/mini-manipulator" [2].

Most researchers are concerned with kinematic properties of the general 6-DOF parallel platform architecture. The design of in-parallel actuated mechanisms with less than 6-DOF is less developed. Besides, some application tasks don't require all 6-DOF of the platform motion. In many respects, the paper focuses on theoretical design based on displacement group theory and fabrication of a 3-DOF linear type tripod mechanism evolved from previous researches and experiences of the authors concerning the design and control of parallel robotic platform manipulators [3]. This mechanism has been studied in-principle by several researchers [1, 5-7], but the nature of motions, which can be generated by the mechanism, is not analyzed in depth. This type of parallel mechanism has the potential to produce high precision, a desirable attribute for a wrist robotic device. Hence, the ideal application of such special parallel mechanism is a micro-motion manipulator [1]. The major goals of this research supported by NgeeAnn Polytechnic are:

- Development of the technology for new generation of robot systems carrying out dexterous manipulation tasks.
- To train Mechanical Engineering staff and students in kinematics modeling and motion control of closed-loop chain mechanisms attracting more and more attention as a competent device for robotics and automation.
- To support research and promote interest and skill development in robot manipulator design and control issues.

In this paper, the mechanical generators of Lie subgroups of Euclidean displacement, which have proved to be a useful tool to model displacements of a mechanism, are first introduced. Then the formula of modified Grüebler mobility allowing to calculate the theoretical number of degrees-of-freedom within a mechanism is presented. Next, the equivalent kinematic chains of a family of mechanical generators of a planar-spherical bond can be derived. Finally, as an application of the above method the mechanical design of a 3-DOF 3-*RPS* tripod belonging to this family is described for model technology demonstration.

2 MECHANICAL GENERATORS OF LIE SUBGROUPS OF DISPLACEMENTS

The algebraic structure of Lie group of the set of Euclidean displacements {D} is a fundamental tool in the analysis of general properties of mechanisms. The comprehensive list of Lie subgroups of {D} is given in

[8, 9]. For sake of succinctness, only four exemplary Lie subgroups of dimension less than or equal to three, namely planar displacements, rotations, linear translations and spherical motions, are explained here. Using intrinsic geometrical entities instead of coordinates and components in a given frame of reference, the previous subgroups can be denoted as {G (Pl)}, {R (N, **u**)}, {T (**v**)}, {S (N)}. Curly brackets are used for indicating sets. The capital characters G, R, T, S designate a type of motion, more precisely a class of conjugacy employing the vocabulary of group theory. {G (Pl)} means planar gliding parallel to the plane Pl representing the direction of a plane of the 3-dimensional Euclidean affine space. One can write also {G (**w**)} where **w** is a unit vector perpendicular to the plane Pl. {R (N, **u**)} holds for rotations around the axis determined by the pair (N, **u**) where N is a point and **u** a unit vector, {T (**v**)} for translations parallel to the given unit vector **v** and {S (N)} for spherical motions around the point N. **u** is a unit vector of the 3-dimensional Euclidean vector space, and N is a point of the 3-dimensional Euclidean affine space. Hence all the theorems of the group theory can be used to establish the mechanical properties of mechanisms.

The set of allowed relative displacements between two rigid bodies, which belong to a given kinematic chain, is called mechanical bond. When a mechanical bond is given, a kinematic chain, which generates the bond will be named mechanical generator of the bond. The cases of mechanical bonds that are not manifolds of the displacement group {D} are out of the scope of this paper. Bonds, which are Lie subgroups of {D}, play a key role in the theory of mechanisms. A very important property of a subgroup is the closure of the product: the product of two elements of a given subgroup belongs to this subgroup. A serial arrangement of two mechanical generators of two bonds produces a new bond between the first rigid body and the end body and the new bond is the product of the serialized bonds.

As an example, let us consider a sequence of two revolute pairs *R* and a prismatic pair *P*. The first *R* pair produces the subgroup {R (M, **u**)}, the second *P* pair the subgroup {T (**v**)}, and the last *R* pair the subgroup {R (N, **w**)}. M is a point on the first axis of rotation, **u** is a unit vector parallel to this axis, **v** is a unit vector parallel to the translation direction, N is a point on the second axis of rotation, and **w** is a unit vector parallel to this axis. The two points M and N are distinct. The bond between the first body and the end body, is given by the following product:

{R (M, **u**)•T (**v**)•R (N, **w**)} = {R (M, **u**)}•{T (**v**)}•{R (N, **w**)}, where • is a symbol for the product of transformations. Generally {R (M, **u**)}•{T (**v**)}•{R (N, **w**)} is not a subgroup and is just a 3-dimensional manifold of {D}. In a possible special case the two *R* pairs are parallel: **u** = **w** and **v** is perpendicular to **u**, then the subgroups {R (M, **u**)}, {R (N, **u**)} and {T (**v**)} are included in the subgroup {G (**u**)} of planar motions, which combines rotations {R (M, **u**)} and {R (N, **u**)} about the axis (M, **u**), respectively (N, **u**) with translations {T (**v**)} parallel to **v**. We can write:

{R (M, **u**)} ⊆ {G (**u**)}
{R (N, **u**)} ⊆ {G (**u**)}
{T (**v**)} ⊆ {G (**u**)}

This implies {R (M, **u**)}•{T (**v**)}•{R (N, **u**)} ⊆ {G (**u**)} because of the product closure in the subgroup {G (**u**)}.

{R (M, **u**)}•{T (**v**)}•{R (N, **u**)} is a 3-dimensional manifold included in the 3-dimensional subgroup {G (**u**)}. Considering only motion types and neglecting the possible difference of motion amplitude, we can say that the sequence *RPR* is a mechanical generator of the subgroup {G (**u**)} or {G (Pl)} if the two *R* axes are parallel to **u**, i.e. perpendicular to the plane Pl, and the *P* direction is perpendicular to **u**:

{R (M, **u**)}•{T (**v**)}•{R (N, **u**)} = {G (**u**)} **(1)**

Equation **(1)** can be illustrated by Fig. 1, which shows the mechanical generator of a planar subgroup {G (Pl)} obtained by a sequence *RPR* of lower kinematic *R*, *P* and *R* pairs between the bodies A and B.

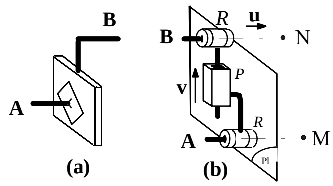


Fig. 1 Generator of subgroup {G (Pl)}

The other generators of planar gliding motion {G (**u**)} are readily obtained through the same reasoning and they are *RRP*, *RRR*, *PRP, PPR,* provided the *R* pair axes are parallel to **u** and the *P* pairs perpendicular to **u** (Fig. 2). The ordering inversion of a sequence, for example *PRR,* remains a generator of {G (**u**)}.

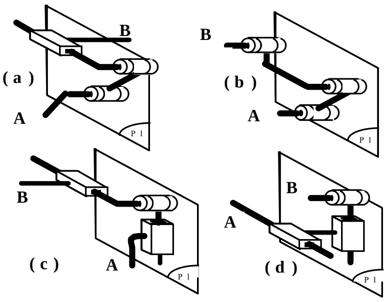


Fig. 2 Other generators of subgroup {G (Pl)}

In the same manner, one can prove that a sequence of three revolute pairs *RRR* with intersecting axes is a mechanical generator of the subgroup {S (N)}, N being the common point of the intersecting axes. Schematic drawing of the corresponding equivalent kinematic chain is shown in Fig. 3.

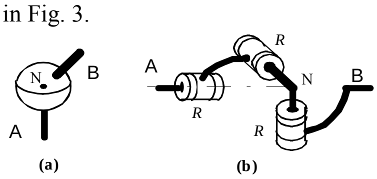


Fig. 3 Generator of subgroup {S (N)}

In order to obtain all the mechanical bonds of a kinematic chain, from the given mechanical bonds of the kinematic pairs, two operations between the mechanical bonds are used. There are the composition and the intersection of bonds. The composition of bonds results of the ordering in series of two bonds. The resulting subset of possible displacements is the composition product of the subsets, which are associated to the bonds in series. The intersection of mechanical bonds is obtained when two bodies are connected in parallel by two mechanical bonds. In this case there is a closed loop of the mechanism. The intersection bond is obtained by taking the intersection set of subsets that represent the in-parallel bonds. So, group theory provides useful theorems that implement the composition and intersection of mechanical bonds. In a subgroup, the composition product is a closed product. The intersection set of two subgroups is always a subgroup.

3 MOBILITY ANALYSIS OF A GENERAL 3-DOF TRIPOD MECHANISM

By definition, the mobility M of a mechanism is the actual number of degrees-of-freedom in a mechanism. The number of degrees-of-freedom is the number of independent joint variables, which must be specified in order to define the position of all links within a mechanism. A classical way to obtain the mobility of a kinematic chain consists of employing the modified Grüebler's criterion based on the Graphs theory, which calculates the theoretical number of degrees-of-freedom within a mechanism. Let b the number of rigid bodies in the mechanism including the fixed body, n the number of bonds, f_i the number of degrees of freedom for i^{th} bond, E_c the number of independent loops in a kinematic chain or number of kinematic equations, and I_c the unknown kinematic variables (for example joint velocities). If all of the bonds define independent constraints, M is given by

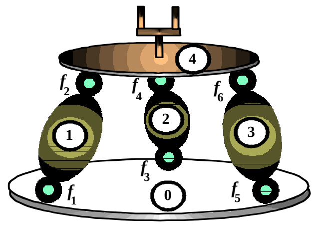
$$M = I_c - E_c \qquad \textbf{(2)}$$

Where $E_c = \lambda\,(n - b + 1)$ and $I_c = \sum_{i=1}^{n} f_i$

$\lambda = 3$ for planar mechanism and 6 for spatial mechanism. Equation **(2)** is called modified Grüebler's criterion. In the Hervé generalized formula [8], λ becomes the dimension of a Lie subgroup of {D} and may be 1, 2, 3, 4 or 5 depending on the possible inclusion of all mechanical bonds in a given Lie subgroup of {D}. For example in planar mechanisms, any bond between any couple of rigid body is included in a subgroup of the type {G (**u**)}, which is a 3-dimensional manifold.

If $M < 0$, the system is hyperstatic, i.e. the mechanism becomes a statically indeterminate structure. If $M = 0$, the system is isostatic, i.e. the mechanism is in equilibrium. Finally if $M > 0$**,** the system is mobile, i.e. if any M kinematic variables between the constituent bodies are known, then all other kinematic variables can be determined. For example, the four bar linkage, which is considered as a basic planar mechanism, has a mobility $M = 1$.

Using this criterion for non over-constrained mechanisms (λ=6), it is easy to prove in Fig. 4 for the case of a general 3-DOF tripod with equal limbs that $I_c = 15$, i.e. each limb of the tripod is endowed 5 degrees of freedom. Otherwise, each limb has to generate a 5-dimensional manifold of the displacement group. In the special case of orientational tripod studied by Karouia and Hervé [10], the 5-dimensional manifold has to include the Lie subgroup {S (N)} of the Lie group {D} of displacements. In a general 3-DOF tripod, the moving platform undergoes a 3-dimensional manifold of screw motions, which can't be reduced into a product of independent sets of pure rotations or pure translations.



$$b = 5, \quad n = 6, \quad \lambda = 6 \;\Rightarrow\; E_c = \lambda\,(n - b + 1) = 12$$

$$M = I_c - E_c = 3 \;\Rightarrow\; I_c = \sum_{i=1}^{n} f_i = 15$$

Fig. 4 Mobility of a general 3-DOF tripod

4 3-DOF TRIPOD MECHANISMS WITH PLANAR-SPHERICAL BONDS

4.1 Generators of a Planar-Spherical Bond

Based on the previous study in sections 2 and 3, we will focus our attention to the special case of tripod, each limb of which generates a 5-dimensional manifold of the displacement group, otherwise a 5-DOF kinematic bond, and this bond can be represented by {G (**u**)}•{S (N)}. This is the product of two subgroups but not a subgroup. This kinematic bond is produced evidently by the serial setting of a planar pair *Pl* and a spherical pair *S* (Fig. 5(a)) and therefore can be called planar-spherical bond.

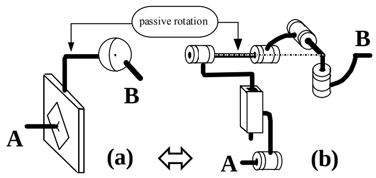


Fig. 5 planar-spherical bond

However the bond representation {G (**u**)}•{S (N)} use 6 parameters, 3 for {G (**u**)} and 3 for {S (N)}, whereas 5 parameters are enough to characterize any 5-dimensional bond. The superfluous parameter can be eliminated by considering the intersection {G (**u**)}∩{S (N)}, which can be readily proven equal to {R (N, **u**)}. {R (N, **u**)} is the subgroup of rotations around the unit vector **u**, which is perpendicular to the plane Pl drawn from point N. These rotations are passive in the bond. In the following we will find two families of generators, which don't have passive mobility. Because of the product closure in the subgroup {G (**u**)}, we can write

{G (**u**)} = {R (M, **u**)}•{T (**v**)}•{R (N, **u**)} **(3)**

where M and N are distinct points, **v** is a unit vector perpendicular to the unit vector **u**. Also, because of the product closure in the subgroup {S (N)}, we can write

{S (N)} = {R (N, **u**)}•{R (N, **i**)}•{R (N, **j**)} **(4)**

where N is the common point of the intersecting axes, and the vectors **u**, **i** and **j** are linearly independent. A possible representation of the mechanical bond {G (**u**)}•{S (N)} is as follows.

{G (**u**)}•{S (N)} = {R (M, **u**)}•{T (**v**)}•{R (N, **u**)}•
{R (N, **u**)}•{R (N, **i**)}•{R (N, **j**)} **(5)**

Equation **(5)** is illustrated by Fig. 5(b).
Because of the product closure in the subgroup $\{R\ (N, \mathbf{u})\}$, we can write:

$$\{R\ (N, \mathbf{u})\}\bullet\{R\ (N, \mathbf{u})\} = \{R\ (N, \mathbf{u})\}^2 = \{R\ (N, \mathbf{u})\}.$$

Equation **(5)** becomes:

$$\begin{aligned}\{G\ (\mathbf{u})\}\bullet\{S\ (N)\} &= \{R\ (M, \mathbf{u})\}\bullet\{T\ (\mathbf{v})\}\bullet[\{R\ (N, \mathbf{u})\}\bullet \{R\ (N, \mathbf{i})\}\bullet\{R\ (N, \mathbf{j})\}] \\ &= \{R\ (M, \mathbf{u})\}\bullet\{T\ (\mathbf{v})\}\bullet\{S\ (N)\} \qquad \textbf{(6)}\end{aligned}$$

The passive mobility is eliminated in this new representation of the given bond. This first way for obtaining a regular generator of a planar-spherical bond is explained by Fig. 6(b).

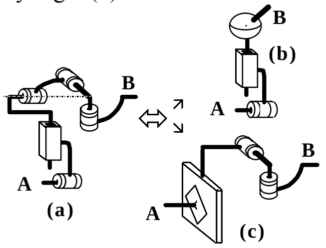


Fig. 6 Two families of generators of a planar-spherical bond

Equation **(6)** leads to a limb chain, namely the family *<u>RP</u>(S)* illustrated in Fig. 6(a). In this kinematic chain, the *P* pair direction is perpendicular to the axis of *R* pair. The underline denotes a planar system and the parenthesis denotes a spherical system. Similarly, we can find other mechanical generators of the same bond $\{G\ (\mathbf{u})\}\bullet\{S\ (N)\}$ with the following list of possible limbs: *<u>RR</u>(S)*, *<u>PR</u>(S)* and *<u>PP</u>(S)*, where (*S*) can be replaced by a sequence of (*RRR)*. The above-described family of equivalent kinematic chains is made of sequences of a 2-DOF planar system and a 3-DOF spherical motion generator. This family can be designated by one of its representative, namely *<u>RP</u>(S).*
Rewriting Eq. **(5)** as follows.

$$\begin{aligned}\{G\ (\mathbf{u})\}\bullet\{S\ (N)\} &= [\{R\ (M, \mathbf{u})\}\bullet\{T\ (\mathbf{v})\}\bullet\{R\ (N, \mathbf{u})\}] \bullet\{R\ (N, \mathbf{i})\}\bullet\{R\ (N, \mathbf{j})\} \\ &= \{G\ (\mathbf{u})\}\bullet\{R\ (N, \mathbf{i})\}\bullet\{R\ (N, \mathbf{j})\} \qquad \textbf{(7)}\end{aligned}$$

Equation **(7)** is another way to eliminate the passive mobility represented by $\{R\ (N, \mathbf{u})\}$, and leads to another family of equivalent limb chains, namely the family *Pl-(RR),* which generates the same planar-spherical bond (Fig. 6(c)). In this family, the two *R* axes of the notation (*RR*) have to intersect at a point and we can obtain the following list of possible limbs: *<u>RRR</u>(RR)*, *<u>RRP</u>(RR), <u>PRR</u>(RR), <u>RPR</u>(RR), <u>RPP</u>(RR)* and *<u>PPR</u>(RR)*.
Since we are more interested in the design of parallel-mechanism limbs that implement a spherical pair *S*, the family *<u>RP</u>(S)* is only considered for further study (Fig. 7). It is worth noting that spherical pairs can be simply realized employing commercially available spherical plain bearings.

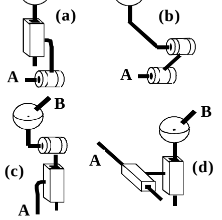


Fig. 7 Limbs of the family *<u>RP</u>(S)*

4.2 Parallel Arrangement of 3-*<u>RP</u>(S)* Tripod

Because of potential applications in assembly automation, we will emphasize the special arrangement *3-<u>RP</u>*(*S*) with three planes that are parallel to a straight line. As a matter of fact, the intersection of two distinct subgroups of planar gliding motions is a subgroup of linear translations, which are parallel to the straight line of the plane intersection. If the third plane is also parallel to this line, linear translations will be produced by the parallel mechanism. Such a translation is useful to insert a peg in a bored hole.
Let us consider three limbs of the structural type *<u>RP</u>*(*S*) connecting a fixed base to a moving platform. The three limbs produce three kinematic bonds $\{G\ (\mathbf{w_1})\}\bullet\{S\ (B_1)\}$, $\{G\ (\mathbf{w_2})\}\bullet\{S\ (B_2)\}$ and $\{G\ (\mathbf{w_3})\}\bullet\{S\ (B_3)\}$. Let us suppose that the third limb is removed for theoretical purpose. The platform can undergo motions that are represented by $\{G\ (\mathbf{w_1})\}\bullet\{S\ (B_1)\} \cap \{G\ (\mathbf{w_2})\}\bullet\{S\ (B_2)\}$.
It is straightforward to prove that
$\{G\ (\mathbf{w_1})\}\bullet\{S\ (B_1)\}\cap\{G\ (\mathbf{w_2})\}\bullet\{S\ (B_2)\}\subseteq \{G\ (\mathbf{w_1})\} \cap \{G\ (\mathbf{w_2})\}$ because the identical transformation E belongs to $\{S\ (B_1)\}$ and $\{S\ (B_2)\}$, and replacing $\{S\ (B_1)\}$ and $\{S\ (B_2)\}$ by E gives a subset of $\{G\ (\mathbf{w_1})\}\bullet\{S\ (B_1)\} \cap \{G\ (\mathbf{w_2})\}\bullet\{S\ (B_2)\}$. We have $\{G\ (\mathbf{w_1})\} \cap \{G\ (\mathbf{w_2})\} = \{T\ (\mathbf{z_0})\}$, $\mathbf{z_0}$ being a unit vector that is perpendicular to the plane ($\mathbf{w_1}$, $\mathbf{w_2}$). We can conclude that the subgroup $\{T\ (\mathbf{z_0})\}$ is included in the manifold $\{G\ (\mathbf{w_1})\}\bullet\{S\ (B_1)\} \cap \{G\ (\mathbf{w_2})\}\bullet\{S\ (B_2)\}$, which is not a subgroup but a 4-dimensional manifold of {D}.
If $\{G\ (\mathbf{w_1})\} \cap \{G\ (\mathbf{w_3})\}$ is also equal to $\{T\ (\mathbf{z_0})\}$, then $\{G\ (\mathbf{w_2})\} \cap \{G\ (\mathbf{w_3})\}$ will be equal to $\{T\ (\mathbf{z_0})\}$. The whole tripod will produce platform 3-DOF motions represented by
$\{G\ (\mathbf{w_1})\}\bullet\{S\ (B_1)\} \cap \{G\ (\mathbf{w_2})\}\bullet\{S\ (B_2)\} \cap \{G\ (\mathbf{w_3})\}\bullet\{S\ (B_3)\}$, which includes $\{G\ (\mathbf{w_1})\} \cap \{G\ (\mathbf{w_2})\} \cap \{G\ (\mathbf{w_3})\} = \{T\ (\mathbf{z_0})\}$.
We can conclude that the moving platform has translational 1-DOF motion parallel to $\mathbf{z_0}$. The remaining 2-DOF motion set is made of screw motions. Any screw motion being a combination of translation and rotation, the platform can rotate with 2-DOF but generally the allowed rotations are not pure and therefore imply dependent translations.

Any rotation (or screw motion) having an axis parallel to $\mathbf{z_0}$ is forbidden. Let us consider a virtual revolute pair connecting the base to the moving platform. This *R* pair generates the bond $\{R\ (P, \mathbf{z_0})\}$, P being any point, and the motion set of the platform will become (set operation $\cap$ is associative):
$\{G\ (\mathbf{w_1})\}\bullet\{S\ (B_1)\}\cap\{G\ (\mathbf{w_2})\}\bullet\{S\ (B_2)\}\cap\{G\ (\mathbf{w_3})\}\bullet\{S\ (B_3)\} \cap \{R\ (P, \mathbf{z_0})\}$
The above-written intersection is included in the partial intersection $\{G\ (\mathbf{w_1})\}\bullet\{S\ (B_1)\} \cap \{R\ (P, \mathbf{z_0})\}$. This last intersection of two kinematic bonds symbolizes a single-loop chain. Neglecting the 1-DOF passive mobility in $\{G\ (\mathbf{w_1})\}\bullet\{S\ (B_1)\}$, the addition of DOF in the loop is 6, hence generally the single loop is a static structure and $\{G\ (\mathbf{w_1})\}\bullet\{S\ (B_1)\} \cap \{R\ (P, \mathbf{z_0})\} = \{E\}$, {E} being the improper subgroup that contains only the identical transformation E. Exceptions may happen if $\{S\ (B_1)\} \cap \{R\ (P, \mathbf{z_0})\} \neq \{E\}$ or if $\{G\ (\mathbf{w_1})\} \cap \{R\ (P, \mathbf{z_0})\} \neq \{E\}$. The first exception can occur if and only if the point B_1 belongs to the axis (P, $\mathbf{z_0}$), the second exception

if and only if $\mathbf{z_0}$ is parallel to $\mathbf{w_1}$. If these geometrical conditions are effective for one limb, they can't be achieved in the other two limbs when (B_1, B_2, B_3) is a true triangle and the vectors $\mathbf{w_1}$, $\mathbf{w_2}$ and $\mathbf{w_3}$ are distinct.

Let us suppose now that {R (P, $\mathbf{z_0}$)} is replaced by {R (P, $\mathbf{i}$)}, where $\mathbf{i}$ is an unknown unit vector and P an unknown point. In a special configuration of whole parallel mechanism the straight line B_1B_2 may be parallel to $\mathbf{w_3}$. If $\mathbf{i}$ is chosen equal to $\mathbf{w_3}$ and P on the line B_1B_2 (for example P = B_1), the above-mentioned conditions of exceptional mobility are obtained at three times because we have

1. $\{S(B_1)\} \cap \{R(B_1, \mathbf{i})\} = \{S(B_1)\} \cap \{R(B_1, \mathbf{w_3})\} = \{R(B_1, \mathbf{w_3})\}$
2. $\{S(B_2)\} \cap \{R(B_2, \mathbf{w_3})\} = \{R(B_2, \mathbf{w_3})\} = \{R(B_1, \mathbf{w_3})\}$
3. $\{G(\mathbf{w_3})\} \cap \{R(B_1, \mathbf{w_3})\} = \{R(B_1, \mathbf{w_3})\}$

The subgroup {R (B_1, $\mathbf{w_3}$)} is included in the three bonds generated by the three limbs and therefore included in the three bonds intersection. The moving platform can rotate around the axis of the straight line B_1B_2. If the mechanism has a ternary axis of symmetry, which is parallel to the three planes of the three planar-spherical bonds, like the prototype of the following paragraph, (B_1, B_2, B_3) is an equilateral triangle, whose edges are three axes of allowed pure rotations. Other lines can't be axes of pure rotation.

In order to explain again the platform motion in an intuitive manner, we may remark that the planar-spherical bond can be obtained by a simple contact of a sphere on a plane. Looking at figure 5(a) we notice that the sphere size has no importance and that the plane is actually a plane direction and therefore can be replaced by a parallel plane. Hence the sphere can get in touch with the plane and then the contact is maintained. The possible platform motions are those of a body having three spheres, which remain in contact with three fixed planes that are parallel to a line. The following Figure 8 shows this theoretical device. It is worth noting that the spheres can have a zero diameter and therefore can be reduced into points.

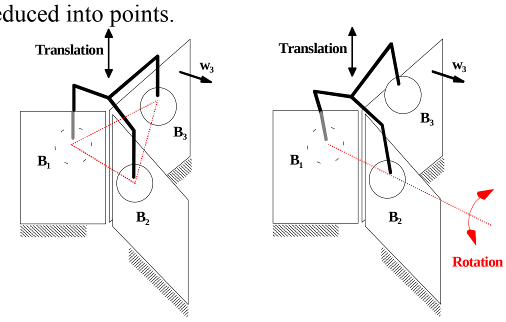


Fig. 8 Equivalent kinematic chains of 3-DOF 3-*RPS* tripods

5 MECHANICAL DESIGN OF A 3-DOF 3-*RPS* TRIPOD

By adopting the *RPS* tripod architecture, our designed prototype is a linear type 3-DOF parallel manipulator represented in Fig. 9. It composes a mobile platform end-effector and a fixed base plate, connected by three variable length links representing three actuated prismatic joints, which are based on ball-screw system. Each of the three ball-screws is coupled to a base plate through a passive revolute joint, and to the end-effector attachment plate through a nut which is free to rotate about three perpendicular axes by virtue of a spherical joint coupling, which is customized using the THK spherical plain bearing type SA1 from Japan. Since the manipulator in this project is intended to be for general purpose, the base and mobile plates are both of equilateral triangle shape. The entire arrangement of attachment points on mobile and base plates is made as symmetric as possible to simplify analysis and operation. This mechanism has three degrees of freedom: two for orientation (pitch and yaw rotations) and one for translation freedom (plunging motion), and can provide the necessary flexibility for insertion operations with accuracy.

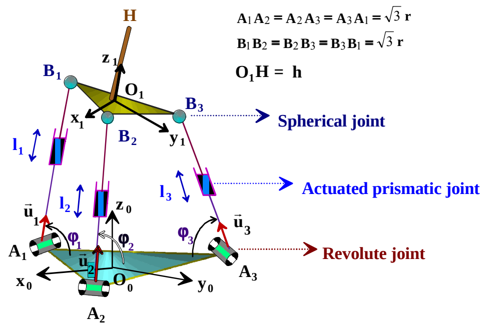


Fig. 9 A linear type 3-DOF parallel mechanism

Our design objective is to obtain the smallest size of the mechanism based on commercially available mini-motor with specified torque. Using inverse kinematics described in [5], the workspace volume can be estimated by limiting the motion range of actuators $(\mathbf{l_i}, \boldsymbol{\varphi_i})_{(i=1,2,3)}$. One of the most interesting point in this mechanism design is to select a value $\boldsymbol{\rho}$, representing the ratio between the length of equilateral triangle of end plate a_1 and that of the fixed base plate a_0, which produces a large workspace volume. Based on available size of the three mini-motors, the minimum value of a_0 is 89mm. The case $\boldsymbol{\rho} > 1$ is excluded, because end plate inertia will be higher than the one of base plate, and therefore the dynamic performance characteristics such as high speed and acceleration cannot be obtained. First the volume is estimated by limiting the actuator motion range $\mathbf{l_i}$ (**i** = 1,2,3) (for actuator **i**, $25\text{mm} \le \mathbf{l_i} \le 64\text{mm}$). Then, the volume is refined by taking into account the motion range of the pin joints $\boldsymbol{\varphi_i}$ (**i** = 1,2,3) (for pin joint **i**, $55^\circ \le \boldsymbol{\varphi_i} \le 90^\circ$). Based on computed workspace studied in [5] the 2/3 ratio offers the best performance. But, it was decided to choose $\boldsymbol{\rho}$ as 1 because the individual contribution of each actuator to end plate motion is decoupled, so the kinematics analysis is simplified as much as possible, and therefore the mechanism can be used for real time control.

A prototype parallel mechanism illustrated in Fig. 10 has been designed using Pro-Engineer software, and CNC machined for educational purpose at the NgeeAnn Polytechnic of Singapore. The 3D workspace of the parallel manipulator when $\boldsymbol{\rho} = 1$, is plotted in Fig. 11 by a numerical procedure explained in [4] and inverse kinematics described in [5]. Finally, table 1 shows the specification of the prototype.

6 CONCLUSION

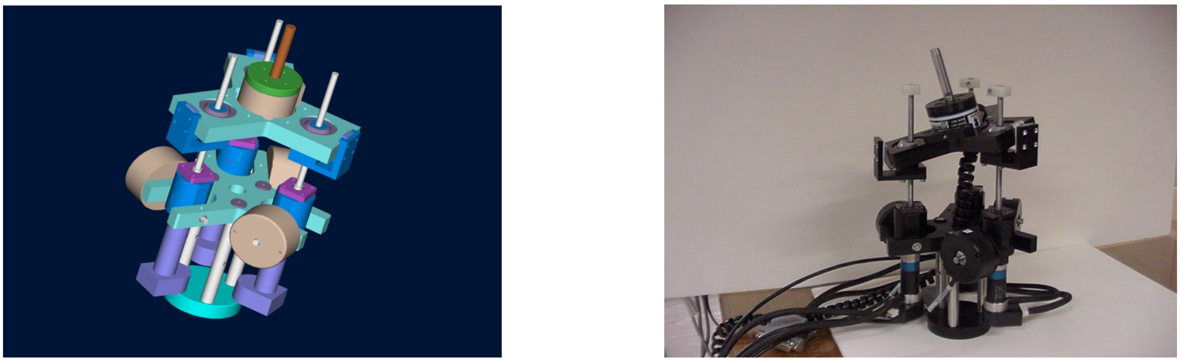
In this paper, equivalent kinematic chains analysis of 3-DOF tripod mechanisms with planar-spherical bonds have been presented using the group theory of displacements and the mobility analysis based on the modified Grüebler's criterion, which are a powerful tool for the motion analysis and synthesis of parallel mechanisms. New design of parallel robotic systems can be obtained thanks to this method.

Applying the above method and based on the simulation study of workspace volume using inverse kinematics, a prototype 3-*RPS* parallel mechanism has been designed and fabricated with its integrated control system for model technology demonstration and for educational purpose at the NgeeAnn Polytechnic of Singapore.

Future research works will be carried out on the development of kinematics hybrid position-force control software for interactions between the parallel manipulator and hard contact environment.

REFERENCES

1. Roth B., Waldron K. J., Raghavan M., "Kinematics of a Hybrid Series-Parallel Manipulator System", ASME *Journal of Dynamic Systems, Measurement, and Control*, Vol.111, 1989, pp.211-221.
2. Khatib O., "Reduced Effective Inertia in Macro-/Mini-Manipulator Systems", *Robotics Research 5*, Miura H. and Arimoto S., Eds., Cambridge: MIT press, 1990, pp.279-284.
3. Huynh P. et al., "Optimal Velocity Based Control of a Parallel Manipulator with Fixed Linear Actuators", *Proceedings of the IEEE/RSJ International Conference on Intelligent Robot and Systems*, Grenoble, France, Sept. 7-11,1997, pp.1125-1130.
4. Huynh P., "Kinematic Performance Evaluation of Linear Type Parallel Mechanisms", *Sixth International Conference on Control, Automation, Robotics and Vision (ICARCV2000)*, Singapore, Dec. 5-8, 2000.
5. Huynh P. et al., "Kinematics Analysis and Mechatronics System Design Of a 3-DOF In-Parallel Actuated Mechanism", *Seventh International Conference on Control, Automation, Robotics and Vision (ICARCV2002)*, Singapore, Dec. 2-5, 2002.
6. Lee K. M. and Shah D. K., "Kinematic Analysis of a Three-Degrees-of-Freedom In-Parallel Actuated Manipulator", *Proceedings of the International Conference on Robotics and Automation*, Raleigh, USA, 1987, Vol.1, pp.345-350.
7. Kim H. S. and Tsai L. W., "Kinematic Synthesis of Spatial 3-*RPS* Parallel Manipulators", *Proceedings of DETC'02, ASME 2002 Design Engineering Technical Conference and Computers and Information in Engineering Conference*, Montreal, CANADA, Sept. 29-Oct. 02, 2002.
8. Hervé J. M., "Analyse structurale des mécanismes par groupe des déplacements", *Mechanism and Machine Theory*, Vol.17, pp.437-450, 1978.
9. Hervé J. M., "The Lie group of rigid body displacements, a fundamental tool for mechanism design", *Mechanism and Machine Theory*, Vol.34, pp.719-730, 1999.
10. Karouia M. and Hervé J. M., "A Family of Novel Orientational 3-DOF Parallel Robots", *Ro.Man.Sy 2002*, July 1-4, 2002, Udine, Italy.



Pro/E Design **CNC Fabrication**

Fig. 10 3-DOF 3-*RPS* tripod actuated mechanism

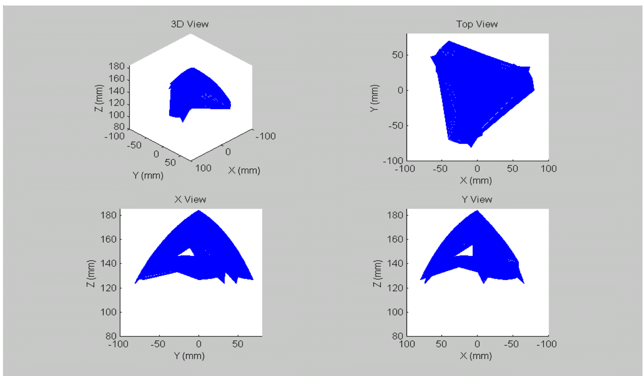


Fig. 11 Workspace volume estimation (Volume = 0.2 dm^3)

	Range	Units
Ball-screw stroke	Min: +25 Max: +64	mm
Total length of ball-screw	120	mm
Pitch of ball-screw	2π	rad/mm
Length of equilateral triangle	89	mm
Height of tool center point	90	mm
Weight	3	kg
Load capacity	3	kg
DC motor output Max. Torque	0.5	Nm
Encoder resolution	2500	ppr
Workspace volume	0.2	dm^3
Resolution of motion: Z	11	µm
Resolution of motion: X, Y	34	µm
Velocity (max) of the end effector	0.013	m/s

Table 1 Specification of the prototype