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LASERS AND THEIR APPLICATIONS

Soliton Laser: Geometry and Stability

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Abstract—It is shown that by properly choosing the geometry of mirrors and the arrangement of nonlinear elements in a confocal nonlinear optical microresonator with gain and losses, spatially localized wave structures, which are stable with respect to a broad class of perturbations, can be excited. *© 2000 MAIK "Nauka/Interperiodica".*

The particle-like excitations of an electromagnetic field in a nonlinear optical resonator—autosolitons [1]—are currently extensively studied because they permit the digital processing and image storage by purely optical methods. Autosolitons have been studied in bistable interferometers [1, 2], lasers with saturable absorption [3], and parametric amplifiers [4]. In a plane-parallel Fabry–Perot resonator with external pumping and losses through mirrors, both autosolitons [1–3] and solitons obtained from the nonlinear Schrödinger equation (filaments) [5] appear, as a rule, against the quasi-homogeneous background whose intensity changes from a few units to hundreds of a percent of the intensity in the autosoliton maximum. In the case of optical data processing and storage, excitation of solitons without a background is preferable, because the energy spent for recording one bit can be substantially lower. The possibility of excitation of solitons of the type sech(x) in a confocal resonator with thin-layer nonlinear elements and the relation between this effect and the known model of longitudinal mode locking [6] have been first discussed in [7]. Such a resonator is called a self-imaging resonator, because in the absence of diaphragms and nonlinear elements an arbitrary image is reconstructed in it from transit to transit due to diffraction (Fig. 1). Later, soliton-like excitations were observed in a self-imaging resonator in [8]. As an amplifying medium and a saturable absorber, thin cells (of thickness smaller than 1 mm) containing a dye and placed in the Fourier-conjugated planes of the resonator were used. Recently, a comparison was made with the results of numerical experiment [9, 10], and a hysteresis was observed upon excitation of solitons. In accordance with the model suggested in [7], the initial set of Maxwell–Bloch equations was split in [9, 10] into a sequence of discrete images of the transverse structure of the field $E_n(x)$—local nonlinear transformations in thin-layer nonlinear elements $f\{E_n\}$ and nonlocal linear operators $\hat{\mathrm{Fr}}$ (Fresnel–Kirchhoff integrals—for intervals of free diffraction of radiation between nonlinear elements and mirrors:

$$E_{n+1}(x) = \hat{\mathrm{Fr}} f\{E_n(x)\},$$
$$E_{n+1}(x_3) = \int K(x_1, x_3) f\{E_n(x_1)\} dx_1. \quad (1)$$

Such an approach generalizes the Fox–Lie method for finding the eigenfunctions of the resonator [11]. As shown in [7], the resonator geometry, i.e., the form and mutual arrangement of mirrors, diaphragms, and nonlinear elements, determine the structure of the kernel K of a linear operator. In particular, the kernel K for the resonator with confocal mirrors and nonlinear elements located in the Fourier-conjugated planes is a real function. When a change in the field E for a round trip in the resonator is small and the width d of the amplifying layer is sufficient, i.e., $G = G_0\{1 - x^2/d^2\}$, the equation of motion of the spatial structure $E_n(x)$ are reduced to the diffusion equation with a nonlinear source [7, 12]:

$$E_{n+1}(x_1) = G_0 f_{\mathrm{abs}}\{E_n(x_1)\} + G_0(4f/kd)^2 \frac{\partial^2 E_n}{\partial x^2},$$
$$f_{\mathrm{abs}}\{E_n\} = (\alpha - 1)E(1 - \beta E^2) + E, \quad (2)$$
$$G_0\alpha < 1, \quad \beta = \sigma T_1,$$

where f is the focal distance, k is the wave number, α is the unsaturated absorption, σ is the stimulated absorption cross section, and T_1 is the longitudinal relaxation time. The diffusion term appears in equation (2) because of the spatial filtration [1, 7] of high spatial frequencies by a transversely inhomogeneous amplifying medium. It is commonly assumed that these equations have solutions in the form of travelling fronts [13]. However, in the case of the nonlinearity of a certain type and a certain diffusion coefficient in front of the second derivative, the solutions in the form of solitary waves can exist [7, 12]. Let us analyze a stability of the exact solution obtained in [12], which describes a single autosoliton in the resonator described in [8–10].

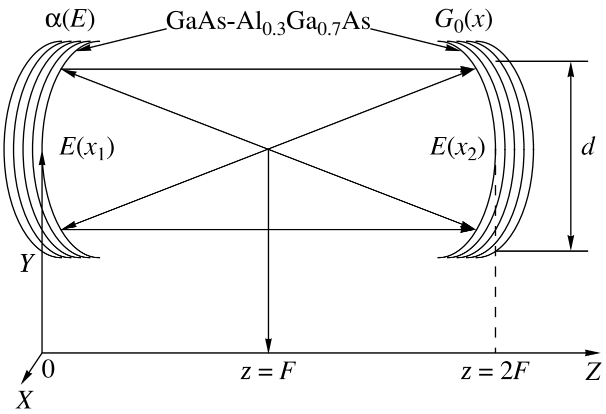

Fig. 1.

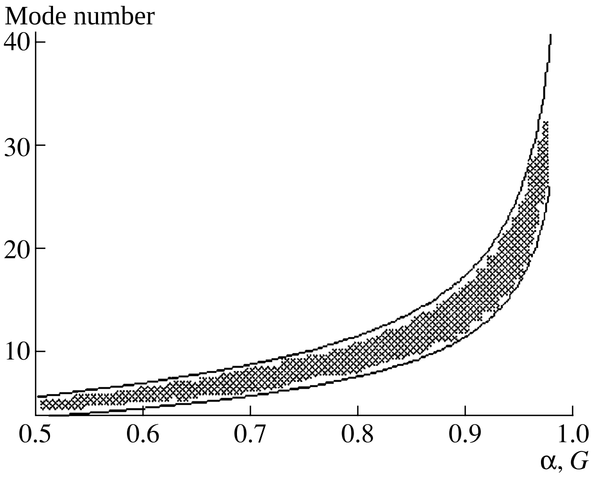

Fig. 2.

This stationary solution of equation (2) in the absorber plane (Fig. 1) has the form [12]

$$E_{n+1}(x) = E_n(x) = S(x),$$
$$S(x) = \sqrt{\frac{2}{\sigma T_1}}\operatorname{sech}\left(\frac{xkd}{4F}\sqrt{\frac{1-\alpha G_0}{G_0}}\right). \quad (3)$$

In the amplifier plane (Fig. 1), the field $S^*(x)$ is described by the Fourier transform

$$S^*(x_2) = \sqrt{\frac{ik}{2\pi F}}\int_{-\infty}^{\infty} S(x_1)\exp[-ik\{x_1x_2\}/F]dx_1,$$
$$S^*(x_2) = \sqrt{\frac{\pi^2 G_0}{2(1-\alpha G_0)\sigma T_1}}\operatorname{sech}\left(\frac{2\pi x_2}{d}\sqrt{\frac{G_0}{1-\alpha G_0}}\right). \quad (4)$$

To study the stability of the solution (3), we pass from the discrete time n, which is measured in the number of round trips in the resonator, to the continuous time t:

$$\frac{\partial E(x)}{\partial t} = G_0 f_{\text{abs}}\{E(x)\} + G_0(4f/kd)^2\frac{\partial^2 E}{\partial x^2}. \quad (5)$$

Let us represent the solution of (5) as a sum $E(x, t) = S(x) + \varepsilon(x, t)$ of the stationary solution being analyzed and a small perturbation of the first-order smallness. The perturbation ε satisfies the equation

$$\frac{\partial \varepsilon(x, t)}{\partial t} = -\alpha G_0\varepsilon(1-3\sigma T_1 S^2) + G_0(4f/kd)^2\frac{\partial^2\varepsilon}{\partial x^2}. \quad (6)$$

We will seek the solution of this equation in the form $\varepsilon = \exp(\gamma t)\psi(x)$, which leads to the eigenvalue problem, which is isomorphous to the known energy-level problem for a quantum particle in the potential $\operatorname{sech}^2(x)$ [14]:

$$\gamma\psi(x) = -\alpha G_0\psi[1-3\sigma T_1\operatorname{sech}^2(\nu x)] + G_0(4f/kd)^2\frac{\partial^2\psi}{\partial x^2},$$
$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}[E + U_0\operatorname{sech}^2(\nu x)]\psi = 0, \quad (7)$$
$$\nu = \frac{kd}{4F}\sqrt{\frac{1-\alpha G_0}{G_0}}.$$

For positive energies $E = -(\gamma + \alpha G)$, the eigenvalue spectrum is continuous, and for negative energies, the spectrum is discrete, while the eigenfunctions represent polynomials of degree n (the level number) [14]. At the same time, the perturbation increment γ is negative in the region $(E + \alpha G) > 0$, for example, for perturbations with a continuous spectrum $(E > 0)$, which represent plane waves for $x = +\infty$ and $-\infty$. This means that such noise perturbations always decay. In addition, there is a region of negative energies, which is hatched in Fig. 2, where the increment

$$\gamma = -\alpha G + \frac{1-\alpha G}{4}\left(-1-2n+\sqrt{1+\frac{24\alpha G}{1-\alpha G}}\right)^2 \quad (8)$$

is also negative. The bound states that correspond to these energies are described by polynomials, and they do not destroy an autosoliton. On the contrary, in the region $(E + \alpha G) < 0$, where the increment γ is positive, the energy E is not only negative but also exceeds in modulus αG. Thus, autosolitons of the type (3), (4) in the confocal resonator (Fig. 1) can become unstable under the action of "coherent" perturbations, which represent the eigenfunctions of the problem (7) with the eigenvalues E exceeding in modulus the unsaturated gain αG of the resonator. For the rest of perturbations, in particular, for noise harmonics with a continuous spectrum, a one-dimensional autosoliton (4) is stable. The autosoliton instability caused by "coherent" perturbations of the finite amplitude can be interpreted as a manifestation of the spatial multistability. Indeed, the solution (3), (4) is not unique, because along with a solution in the form of a single soliton corresponding to the motion along the separatrix [7, 12], there exists a continuum of periodic solutions (cnoidal waves) that are described by elliptic functions [7]. The study of these solutions will be presented elsewhere.

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