Appendix to "Passive solar heating system with high-temperature heat store and air-to-air heat exchanger"

David M Delaney, June 10, 2006
MathCad 13 file

Part1: Calculate the heat flow into the living space air from the heat store through a small part (an "element") of the air-to-air heat exchanger wall one foot wide and VEE tall. Assume the grey radiating planes are parallel and of infinite extent.

$VEE \equiv 0.25$ Units: feet

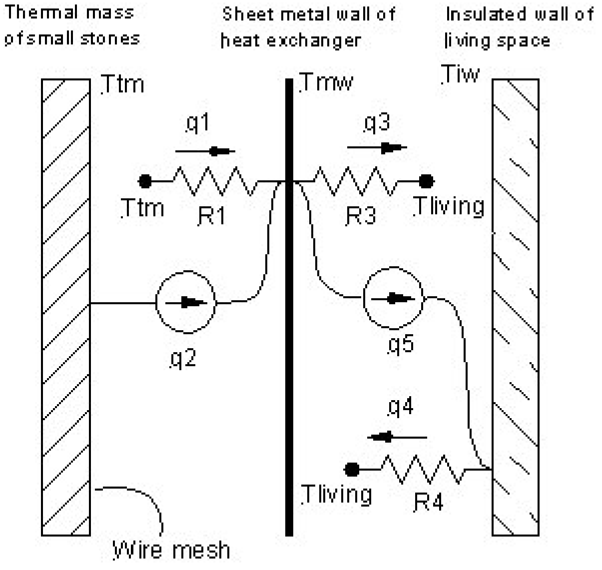
Vertical Element Extent. Calculations are done for a vertical strip of the heat exchanger,
an "element", 1 foot wide and VEE (Vertical Element Extent) high, has an area of VEE square feet.

Thermal resistance of air films, Raf=R1=R3=R4

Units: ft^2.hr.R/Btu

The value of Raf is an important important assumption!

Equivalent circuit for heat flow
from the heat store to the air of the living space
through an element of the heat exchanger.



All temperatures depend on the height of the element

Ttm: temperature of thermal mass and heat store air
Tmw: temperature of the sheet metal wall
Tliving: temperature of living space air in heat exchanger
Tiw: temperature of the south surface of the insulated
wall of the living space
q1: conduction, heat store air to heat exchanger metal wall
q2: radiation, thermal mass to heat exchanger metal wall
q3: conduction, heat exchanger metal wall to house air
q4: conduction, insulated wall of living space to metal wall
q5: radiation, metal wall to insulated wall of living space
qt: total heat transfer to air of living space = q3 + q4

Fig. 5

"hs-he-equiv-circuit-Fig-5.gif"

MathCAD 13 will not allow allow a solve block that solves for variables of different units to be used as the body of a function definition, so I have to careful because MathCAD cannot check dimensional consistency for me. The following solve block (from Given to Find) is the body of the function ElementTempsAndFlows, whose defining assignment statement is at the end of the solve block.

First, solve for the flows and temperatures fpr one foot square of the heat exchanger assuming radiating planes have infinite extent.

Given

All temperatures Rankine

$\sigma \equiv 0.1714\cdot 10^{-8}$ Btu/hr.ft^2.R^4 -- Stefan-Boltzmann constant

Starting guesses:

$\mathrm{Tmw} := 459.67 + 65$ $\quad$ $\mathrm{q1} := 1$ $\quad$ $\mathrm{q2} := 1$

$\mathrm{Tiw} := 459.67 + 60$ $\quad$ $\mathrm{q3} := \mathrm{q1} + \mathrm{q2}$ $\quad$ $\mathrm{q4} := 1$

$\mathrm{q5} := \mathrm{q4}$

Heat flow and heat flow balance equations
for one square foot of the heat exchanger :

$$\mathrm{q1} = \frac{\mathrm{Ttm} - \mathrm{Tmw}}{\mathrm{Raf}}$$

$$\mathrm{q2} = \frac{\sigma\cdot\left(\mathrm{Ttm}^4 - \mathrm{Tmw}^4\right)}{\frac{1}{0.8} + \frac{1}{0.9} - 1}$$

$$\mathrm{q3} = \frac{\mathrm{Tmw} - \mathrm{Tliving}}{\mathrm{Raf}}$$

$$\mathrm{q1} + \mathrm{q2} = \mathrm{q3} + \mathrm{q5}$$

$$\mathrm{q4} = \frac{\mathrm{Tiw} - \mathrm{Tliving}}{\mathrm{Raf}}$$

$$\mathrm{q5} = \frac{\sigma\cdot\left(\mathrm{Tmw}^4 - \mathrm{Tiw}^4\right)}{\frac{1}{0.9} + \frac{1}{0.9} - 1}$$

$$\mathrm{q4} = \mathrm{q5}$$

$\mathrm{ElementTempsAndFlows(Ttm, Tliving, Raf)} := \mathrm{Find(Tmw, Tiw, q1, q2, q3, q4, q5)}$

Tests:

$$\begin{pmatrix} \text{Tmw} \\ \text{Tiw} \\ \text{q1} \\ \text{q2} \\ \text{q3} \\ \text{q4} \\ \text{q5} \end{pmatrix} := \text{ElementTempsAndFlows}(\text{Ttm} \leftarrow 560, \text{Tliving} \leftarrow 540, \text{Raf} \leftarrow 0.6)$$

$$\begin{pmatrix} \text{Tmw} \\ \text{Tiw} \\ \text{q1} \\ \text{q2} \\ \text{q3} \\ \text{q4} \\ \text{q5} \end{pmatrix} = \begin{pmatrix} 550.564 \\ 543.755 \\ 15.726 \\ 8.138 \\ 17.607 \\ 6.258 \\ 6.258 \end{pmatrix}$$

$\text{qt} := \text{q3} + \text{q4}$

$\text{qt} = 23.865$

Define function for heat flow through an element:

$$\text{ElementHeatFlow}(\text{Ttm}, \text{Tliving}, \text{Raf}) := \left| \begin{array}{l} \begin{pmatrix} \text{Tmw} \\ \text{Tiw} \\ \text{q1} \\ \text{q2} \\ \text{q3} \\ \text{q4} \\ \text{q5} \end{pmatrix} \leftarrow \text{ElementTempsAndFlows}(\text{Ttm}, \text{Tliving}, \text{Raf}) \\ \text{return } \ \text{VEE} \cdot (\text{q3} + \text{q4}) \end{array} \right.$$

Units: BTU/hr

$\text{ElementHeatFlow}(560, 540, 0.6) = 5.966$

Units: Btu/(hr*0.25*ft^2)

Part2: Calculate the heating of living space air produced by a vertical slice of the thermal mass and heat exchanger one foot wide.

This calculation makes use of the following approximation for the heating of air: "To raise the temperature of one cubic foot of air by one degree Rankine (or Fahrenheit) in one minute requires a heating power of one Btu per hour". It also assumes a linear rise of the temperature of the thermal mass from TtmBot at the bottom to TtmTop at the top, and that TtmBot = Tinlet, the temperature of the air entering the bottom of the heat exchanger from the bottom of the living space.

```
VerticalSlice(TtmBot, TtmTop, Tinlet, height, flowRate) :=
    NumVertEls ← $\frac{height}{VEE}$
    deltaTtm ← $\frac{TtmTop - TtmBot}{NumVertEls}$
    Ttm ← TtmBot
    Trise ← Tinlet
    totalHeatFlow ← 0
    for hh ∈ 1 .. NumVertEls
        EHF ← ElementHeatFlow(Ttm, Trise, 0.6)
        totalHeatFlow ← totalHeatFlow + EHF
        Trise ← Trise + $\frac{EHF}{flowRate}$
        Ttm ← Ttm + deltaTtm
    return $\begin{pmatrix} totalHeatFlow \\ Trise \end{pmatrix}$
```

Tinlet := 459.67 + 65   Rankine

TtmBot := Tinlet

height := 8

flowRate := 5

$$\begin{pmatrix} \text{totalHeatFlow} \\ \text{Toutlet} \end{pmatrix} := \text{VerticalSlice}(\text{TtmBot}, \text{TtmBot} + 20, \text{Tinlet}, \text{height}, \text{flowRate})$$

$$\begin{pmatrix} \text{totalHeatFlow} \\ \text{Toutlet} \end{pmatrix} = \begin{pmatrix} 54.267 \\ 535.523 \end{pmatrix}$$

Temperatures: Rankine,
heat flows: Btu/hr/horizontal linear foot of heat exchanger

$$\begin{pmatrix} \text{totalHeatFlow} \\ \text{Toutlet} \end{pmatrix} = \begin{pmatrix} 54.267 \\ 535.523 \end{pmatrix}$$

$\text{Tinlet} = 524.67$

Check consistency:

$$\frac{\text{totalHeatFlow}}{\text{Toutlet} - \text{Tinlet}} = 5$$

$\text{flowRate} = 5$ units: cfm

$i := 0..10$

$\text{flowRate}_0 := 0$

$\text{flowRate}_{i+1} := \text{flowRate}_i + 1$

$\text{flowRate}_0 := 0.2$

units: cfm

$$\begin{pmatrix} \text{totalHeatFlow10}_i \\ \text{Toutlet10}_i \end{pmatrix} := \text{VerticalSlice}(\text{TtmBot}, \text{TtmBot} + 10, \text{Tinlet}, \text{height}, \text{flowRate}_i)$$

$\text{Toutlet10} := (\text{Toutlet10} \cdot \text{R}) / {}^\circ\text{F}$

$$\begin{pmatrix} \text{totalHeatFlow20}_i \\ \text{Toutlet20}_i \end{pmatrix} := \text{VerticalSlice}(\text{TtmBot}, \text{TtmBot} + 20, \text{Tinlet}, \text{height}, \text{flowRate}_i)$$

$\text{Toutlet20} := (\text{Toutlet20} \cdot \text{R}) / {}^\circ\text{F}$

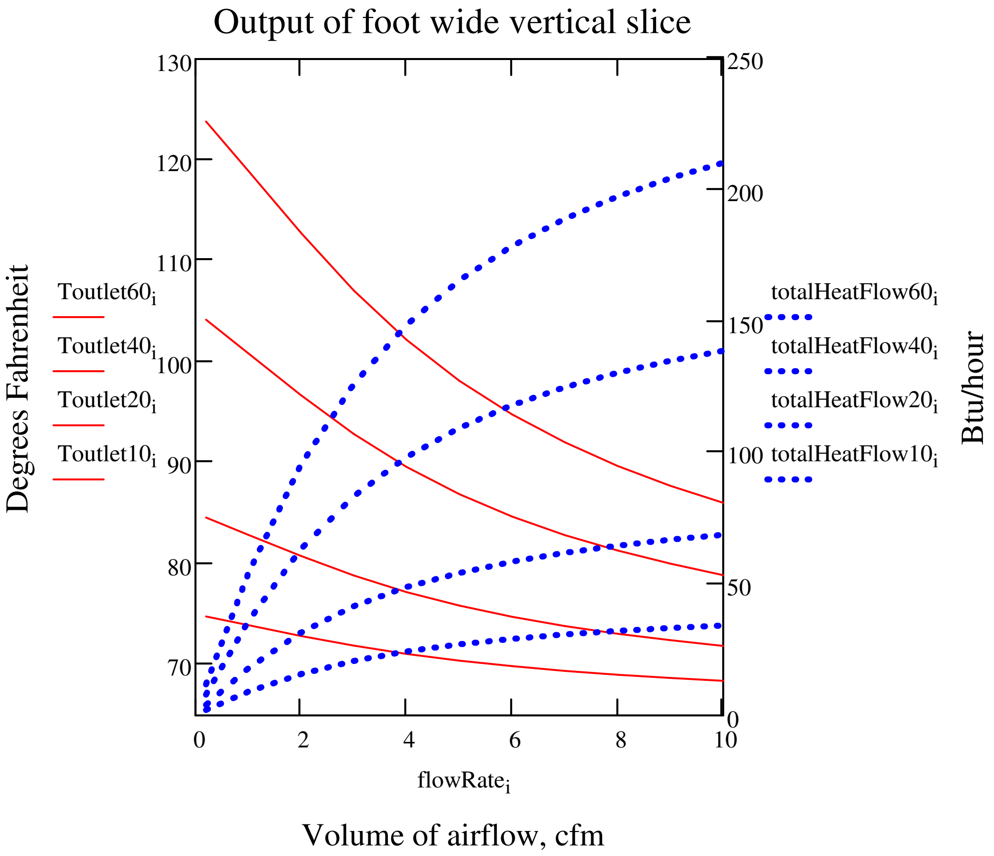
$$\begin{pmatrix} \text{totalHeatFlow40}_i \\ \text{Toutlet40}_i \end{pmatrix} := \text{VerticalSlice}(\text{TtmBot}, \text{TtmBot} + 40, \text{Tinlet}, \text{height}, \text{flowRate}_i)$$

$\text{Toutlet40} := (\text{Toutlet40} \cdot \text{R}) / {}^\circ\text{F}$

$$\begin{pmatrix} \text{totalHeatFlow60}_i \\ \text{Toutlet60}_i \end{pmatrix} := \text{VerticalSlice}(\text{TtmBot}, \text{TtmBot} + 60, \text{Tinlet}, \text{height}, \text{flowRate}_i)$$

$\mathrm{Toutlet60} := (\mathrm{Toutlet60} \cdot \mathrm{R}) / {}^{\circ}\mathrm{F}$

Fig. 6



1) The heating of living space air by a vertical slice of the thermal mass and the heat exchanger.
2) The vertical slice has a horizontal (east-west) extent of one foot.The thermal mass and the heat exchanger have a vertical extent of 8 feet. So does the vertical slice.
3) Output air temperature and air heating power are displayed as a function of the volume rate of flow of living space air through the vertical slice of the heat exchanger in cubic feet per minute.
4) Traces for output air temperature and heating power are displayed for four different temperatures of the top of the thermal mass.
5) The temperature of the thermal mass is assumed to be a linear function of height from 65 degrees Fahrenheit at the bottom of the thermal mass (equal to the temperature of the inlet air from the living space) to the temperature at the top of the thermal mass, increasing nn Fahrenheit degrees in that distance, where nn are the last two characters of the trace name. The temperature of the top of the thermal mass may be read directly from the temperature y intercept of the relevant temperature trace.

6) The traces are in the same vertical order as their names.

Fig. 6

Part3: Compare stratified and unstratified heat stores. The heating of living space air produced by a vertical slice of the thermal mass and heat exchanger one foot wide.

$\mathrm{Tinlet} := 459.67 + 65$ Rankine

$\mathrm{TtmBotUnstrat} := \mathrm{Tinlet} + 10$

$\mathrm{TtmTopUnstrat} := \mathrm{TtmBotUnstrat}$

$\mathrm{TtmBotStrat} := \mathrm{Tinlet}$

$\mathrm{TtmTopStrat} := \mathrm{TtmBotStrat} + 20$

$$\begin{pmatrix} \mathrm{totalHeatFlowUnstrat10}_i \\ \mathrm{ToutletUnstrat10}_i \end{pmatrix} := \mathrm{VerticalSlice}\left(\mathrm{TtmBotUnstrat}, \mathrm{TtmTopUnstrat}, \mathrm{Tinlet}, \mathrm{height}, \mathrm{flowRate}_i\right)$$

$$\mathrm{ToutletUnstrat10}_i := \left(\mathrm{ToutletUnstrat10}_i \cdot R\right) / {}^\circ F$$

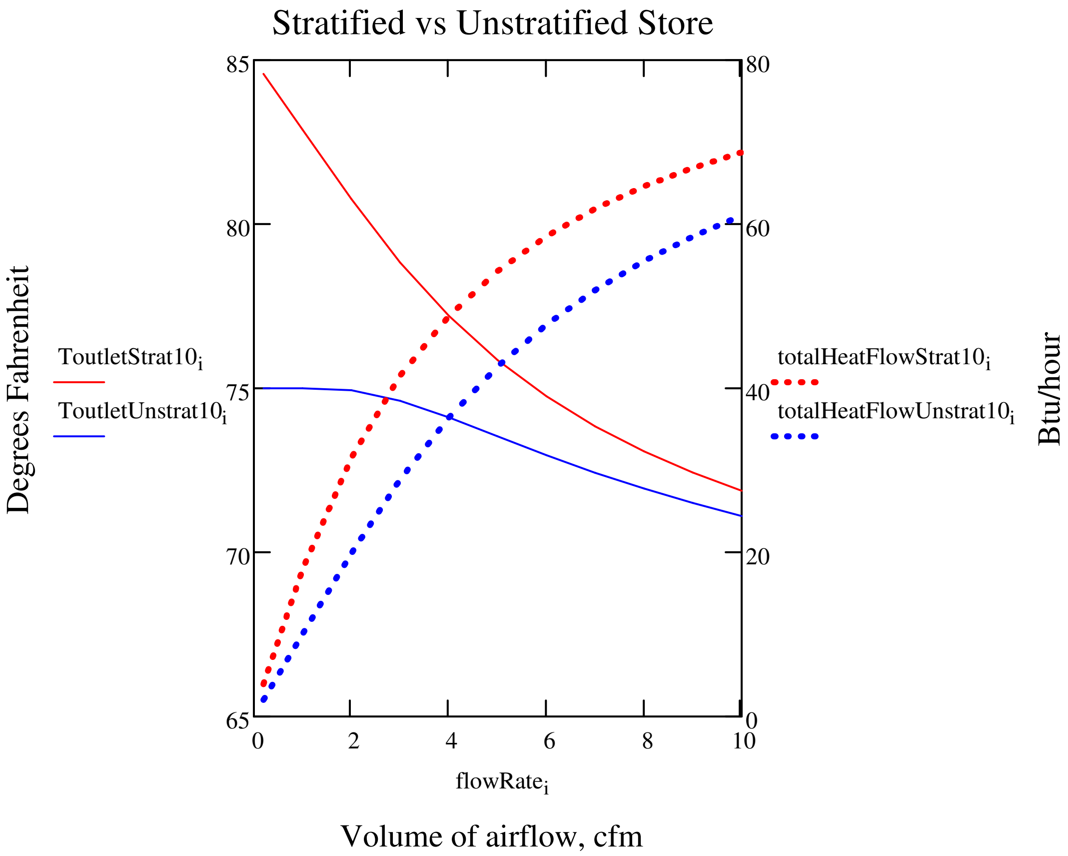
$$\begin{pmatrix} \mathrm{totalHeatFlowStrat10}_i \\ \mathrm{ToutletStrat10}_i \end{pmatrix} := \mathrm{VerticalSlice}\left(\mathrm{TtmBotStrat}, \mathrm{TtmTopStrat}, \mathrm{Tinlet}, \mathrm{height}, \mathrm{flowRate}_i\right)$$

$$\mathrm{ToutletStrat10}_i := \left(\mathrm{ToutletStrat10}_i \cdot R\right) / {}^\circ F$$

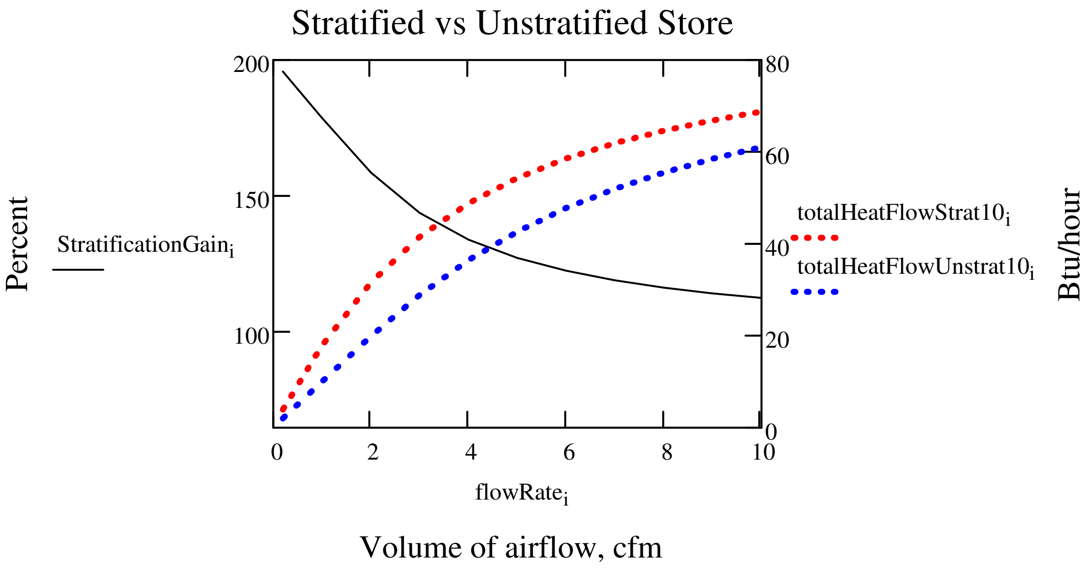
Comparison of stratified and unstratified heat stores

June 12, 2006 David M. Delaney

1) The heating of living space air by a stratified and an unstratified heat store.
2) The heating is computed for a vertical slice of each heat store and its heat exchanger. The slice is 1 foot wide and 8 feet high.
4) Tinlet = 65F is the temperature of the air entering the bottom of the heat exchanger from the living space.
5) The unstratified store has a uniform temperature of 75F = Tinlet + 10F from bottom to top.
6) The bottom temperature of the stratified store is Tinlet = 65F. The top temperature of the stratified store is Tinlet+20F = 85F.
7)The temperature of the stratified heat store rises linearly with height from the bottom of the store to the top of the store.
8)The average temperature of both stores is Tinlet + 10F. Both stores hold the same amount of heat energy



$$\text{StratificationGain}_i := \frac{\text{totalHeatFlowStrat10}_i}{\text{totalHeatFlowUnstrat10}_i} \cdot 100$$


Stratified vs Unstratified Store
Percent
StratificationGain$_i$
200
150
100
0
2
4
6
8
10
flowRate$_i$
80
60
40
20
0
totalHeatFlowStrat10$_i$
totalHeatFlowUnstrat10$_i$
Btu/hour
Volume of airflow, cfm