

40 that is not man is not just' is equivalent to the proposition 'nothing that is not man is just.'

20^b The conversion of the position of subject and predicate in a sentence involves no difference in its meaning. Thus we say 'man is white' and 'white is man.' If these were not equivalent, there would be more than one contradictory to the same proposition, whereas it has been demonstrated that each proposition has one proper contradictory and one only. For of the 5 proposition 'man is white' the appropriate contradictory is 'man is not white,' and of the proposition 'white is man,' if its meaning be different, the contradictory will either be 'white is not not-man' or 'white is not man.' Now the former of these is the contradictory of the proposition 'white is not-man,' and the latter of these is the contradictory of the proposition 'man is white';¹⁰ thus there will be two contradictories to one proposition.

10 It is evident, therefore, that the inversion of the relative position of subject and predicate does not affect the sense of affirmations and denials.

* * *

PRIOR ANALYTICS

* * *

24^b A syllogism is discourse in which, certain things being stated, something other than what is stated follows of necessity

¹⁰ Aristotle really begs the question here, when he states that 'white is not man' is the denial of 'man is white.' Pacius explains that 'man is not white' and 'man is white' are in exactly the same relation each to each as 'white is not man' and 'man is white,' and that therefore 'white is not man' and 'man is not white' are identical. This seems fair, but is in itself sufficient to prove the point at issue at once. The argument of the whole, therefore, is unnecessarily complicated.

from their being so. I mean by the last phrase that they produce 20 the consequence, and by this, that no further term is required from without in order to make the consequence necessary.

I call that a perfect syllogism which needs nothing other than what has been stated to make plain what necessarily follows; a syllogism is imperfect, if it needs either one or more proposi- 25 tions, which are indeed the necessary consequences of the terms set down, but have not been expressly stated as premisses.

That one term should be included in another as in a whole is the same as for the other to be predicated of all of the first. And we say that one term is predicated of all of another, whenever no instance of the subject can be found of which the other term cannot be asserted: 'to be predicated of none' must be 30 understood in the same way.

Every premiss states that something either is or must be or 25^a may be the attribute of something else; of premisses of these three kinds some are affirmative, others negative, in respect of each of the three modes of attribution; again some affirmative and negative premisses are universal, others particular, others 5 indefinite. It is necessary then that in universal attribution the terms of the negative premiss should be convertible, e.g. if no pleasure is good, then no good will be pleasure; the terms of the affirmative must be convertible, not however universally, but in part, e.g. if every pleasure is good, some good must be pleasure; the particular affirmative must convert in part (for if 10 some pleasure is good, then some good will be pleasure); but the particular negative need not convert, for if some animal is not man, it does not follow that some man is not animal.

First then take a universal negative with the terms *A* and *B*. 15 If no *B* is *A*, neither can any *A* be *B*. For if some *A* (say *C*) were *B*, it would not be true that no *B* is *A*; for *C* is a *B*. But if every *B* is *A*, then some *A* is *B*. For if no *A* were *B*, then no *B* could be *A*. But we assumed that every *B* is *A*. Similarly too, if the 20 premiss is particular. For if some *B* is *A*, then some of the *As* must be *B*. For if none were, then no *B* would be *A*. But if some *B* is not *A*, there is no necessity that some of the *As* should not

25 be *B*; e.g. let *B* stand for animal and *A* for man. Not every animal is a man; but every man is an animal.

* * *

25^b After these distinctions we now state by what means, when, and how every syllogism is produced; subsequently we must speak of demonstration. Syllogism should be discussed before
30 demonstration, because syllogism is the more general: the demonstration is a sort of syllogism, but not every syllogism is a demonstration.

Whenever three terms are so related to one another that the last is contained in the middle as in a whole, and the middle is either contained in, or excluded from, the first as in or from a
35 whole, the extremes must be related by a perfect syllogism. I call that term middle which is itself contained in another and contains another in itself: in position also this comes in the middle. By extremes I mean both that term which is itself contained in another and that in which another is contained. If *A* is predicated of all *B*, and *B* of all *C*, *A* must be predicated of
40 all *C*: we have already explained what we mean by 'predicated of all.' Similarly also, if *A* is predicated of no *B*, and *B* of
26^a all *C*, it is necessary that no *C* will be *A*.

But if the first term belongs to all the middle, but the middle to none of the last term, there will be no syllogism in respect of the extremes; for nothing necessary follows from the terms being so related; for it is possible that the first should belong
5 either to all or to none of the last, so that neither a particular nor a universal conclusion is necessary. But if there is no necessary consequence, there cannot be a syllogism by means of these premisses. As an example of a universal affirmative relation between the extremes we may take the terms animal, man, horse; of a universal negative relation, the terms animal, man, stone.
10 Nor again can a syllogism be formed when neither the first term belongs to any of the middle, nor the middle to any of the last. As an example of a positive relation between the extremes take the terms science, line, medicine: of a negative relation science, line, unit.

* * *

It is evident also that all the syllogisms in this figure are 26^b perfect (for they are all completed by means of the premisses 30 originally taken) and that all conclusions are proved by this figure, viz. universal and particular, affirmative and negative. Such a figure I call the first.

Whenever the same thing belongs to all of one subject, and to none of another, or to all of each subject or to none of either, 35 I call such a figure the second; by middle term in it I mean that which is predicated of both subjects, by extremes the terms of which this is said, by major extreme that which lies near the middle, by minor that which is further away from the middle. The middle term stands outside the extremes, and is first in 27^a position. A syllogism cannot be perfect anyhow in this figure, but it may be valid whether the terms are related universally or not.

If then the terms are related universally a syllogism will be possible, whenever the middle belongs to all of one subject and to none of another (it does not matter which has the negative 5 relation), but in no other way. Let *M* be predicated of no *N*, but of all *O*. Since, then, the negative relation is convertible, *N* will belong to no *M*: but *M* was assumed to belong to all *O*: consequently *N* will belong to no *O*. This has already been proved. Again if *M* belongs to all *N*, but to no *O*, then *N* will
10 belong to no *O*. For if *M* belongs to no *O*, *O* belongs to no *M*: but *M* (as was said) belongs to all *N*: *O* then will belong to no *N*: for the first figure has again been formed. But since the negative relation is convertible, *N* will belong to no *O*. Thus it will be the same syllogism that proves both conclusions. 15

It is possible to prove these results also by reduction *ad impossibile*.

* * *

It is clear then from what has been said that if the terms 28^a are related to one another in the way stated, a syllogism results of necessity; and if there is a syllogism, the terms must be so related. But it is evident also that all the syllogisms in this figure

5 are imperfect: for all are made perfect by certain supplementary statements, which either are contained in the terms of necessity or are assumed as hypotheses, i.e. when we prove *per impossibile*. And it is evident that an affirmative conclusion is not attained by means of this figure, but all are negative, whether universal or particular.

10 But if one term belongs to all, and another to none, of a third, or if both belong to all, or to none, of it, I call such a figure the third; by middle term in it I mean that of which both the predicates are predicated, by extremes I mean the predicates, by the major extreme that which is further from the middle, by
15 the minor that which is nearer to it. The middle term stands outside the extremes, and is last in position. A syllogism cannot be perfect in this figure either, but it may be valid whether the terms are related universally or not to the middle term.

* * *

It is clear then in this figure also when a syllogism will be possible and when not, if the terms are related universally. For whenever both the terms are affirmative, there will be a syllogism to prove that one extreme belongs to some of the other; but when they are negative, no syllogism will be possible.

28^b But when one is negative, the other affirmative, if the major is negative, the minor affirmative, there will be a syllogism to prove that the one extreme does not belong to some of the other: but if the relation is reversed, no syllogism will be possible.

* * *

29^a It is evident also that in all the figures, whenever a proper
20 syllogism does not result, if both the terms are affirmative or negative nothing necessary follows at all, but if one is affirmative, the other negative, and if the negative is stated universally, a syllogism always results relating the minor¹ to the major term,² e.g. if *A* belongs to all or some *B*, and *B* belongs

¹ As predicate.

² As subject.

to no *C*: for if the premisses are converted it is necessary that
25 *C* does not belong to some *A*. Similarly also in the other figures: a syllogism always results by means of conversion. It is evident also that the substitution of an indefinite for a particular affirmative will effect the same syllogism in all the figures.

It is clear too that all the imperfect syllogisms are made per-
30 fect by means of the first figure. For all are brought to a conclusion either ostensively or *per impossibile*. In both ways the first figure is formed: if they are made perfect ostensively, because (as we saw) all are brought to a conclusion by means of conversion, and conversion produces the first figure: if they are
35 proved *per impossibile*, because on the assumption of the false statement the syllogism comes about by means of the first figure, e.g. in the last figure, if *A* and *B* belong to all *C*, it follows that *A* belongs to some *B*: for if *A* belonged to no *B*, and *B* belongs to all *C*, *A* would belong to no *C*: but (as we stated) it belongs to all *C*. Similarly also with the rest.

It is possible also to reduce all syllogisms to the univer-
29^b sal syllogisms in the first figure. Those in the second figure are clearly made perfect by these, though not all in the same way; the universal syllogisms are made perfect by converting the negative premiss, each of the particular syllogisms by reduction
5 *ad impossibile*. In the first figure particular syllogisms are indeed made perfect by themselves, but it is possible also to prove them by means of the second figure, reducing them *ad impossibile*, e.g. if *A* belongs to all *B*, and *B* to some *C*, it follows that *A* belongs to some *C*. For if it belonged to no *C*, and belongs to all *B*, then *B* will belong to no *C*: this we know by means
10 of the second figure. Similarly also demonstration will be possible in the case of the negative. For if *A* belongs to no *B*, and *B* belongs to some *C*, *A* will not belong to some *C*: for if it belonged to all *C*, and belongs to no *B*, then *B* will belong to no *C*: and this (as we saw) is the middle figure. Consequently, since
15 all syllogisms in the middle figure can be reduced to universal syllogisms in the first figure, and since particular syllogisms in the first figure can be reduced to syllogisms in the middle figure, it is clear that particular syllogisms can be reduced to universal

20 syllogisms in the first figure. Syllogisms in the third figure, if the terms are universal, are directly made perfect by means of those syllogisms; but, when one of the premisses is particular, by means of the *particular* syllogisms in the first figure: and these (we have seen) may be reduced to the universal syllogisms in the first figure: consequently also the particular syllogisms in the third figure may be so reduced. It is clear then that all syllogisms may be reduced to the universal syllogisms in the first figure.

We have stated then how syllogisms which prove that something belongs or does not belong to something else are constituted, both how syllogisms of the same figure are constituted in themselves, and how syllogisms of different figures are related to one another.

* * *

POSTERIOR ANALYTICS

89^b The kinds of question we ask are as many as the kinds of things which we know. They are in fact four:—(1) whether the connexion of an attribute with a thing is a fact, (2) what is the reason of the connexion, (3) whether a thing exists, 25 (4) what is the nature of the thing. Thus, when our question concerns a complex of thing and attribute and we ask whether the thing is thus or otherwise qualified—whether, e.g., the sun suffers eclipse or not—then we are asking as to the fact of a connexion. That our inquiry ceases with the discovery that the sun does suffer eclipse is an indication of this; and if we know from the start that the sun suffers eclipse, we do not inquire whether it does so or not. On the other hand, when we know the fact we ask the reason; as, for example, when we know that the sun is

being eclipsed and that an earthquake is in progress, it is the 30 reason of eclipse or earthquake into which we inquire.

Where a complex is concerned, then, those are the two questions we ask; but for some objects of inquiry we have a different kind of question to ask, such as whether there is or is not a centaur or a God. (By 'is or is not' I mean 'is or is not, without further qualification'; as opposed to 'is or is not (e.g.) white.')

On the other hand, when we have ascertained the thing's existence, we inquire as to its nature, asking, for instance, 'what, then, is God?' or 'what is man?' 35

These, then, are the four kinds of question we ask, and it is in the answers to these questions that our knowledge consists.

Now when we ask whether a connexion is a fact, or whether a thing without qualification *is*, we are really asking whether the connexion or the thing has a 'middle'; and when we have ascertained either that the connexion is a fact or that the thing *is*—i.e. ascertained either the partial or the unqualified being 90^a of the thing—and are proceeding to ask the reason of the connexion or the nature of the thing, then we are asking what the 'middle' is.

(By distinguishing the fact of the connexion and the existence of the thing as respectively the partial and the unqualified being of the thing, I mean that if we ask 'does the moon suffer eclipse?', or 'does the moon wax?', the question concerns a part of the thing's being; for what we are asking in such questions is whether a thing is this or that, i.e. has or has not this or that attribute: whereas, if we ask whether the moon or night exists, the question concerns the unqualified being of a thing.)

We conclude that in all our inquiries we are asking either 5 whether there is a 'middle' or what the 'middle' is: for the 'middle' here is precisely the cause, and it is the cause that we seek in all our inquiries. Thus, 'Does the moon suffer eclipse?' means 'Is there or is there not a cause producing eclipse of the moon?', and when we have learnt that there is, our next question is, 'What, then, is this cause?'; for the cause through which a thing *is*—not *is this or that*, i.e. has this or that attribute, but 10

OF THE FOUR METHODS OF EXPERIMENTAL INQUIRY

§1. The simplest and most obvious modes of singling out from among the circumstances which precede or follow a phenomenon those with which it is really connected by an invariable law are two in number. One is, by comparing together different instances in which the phenomenon occurs. The other is, by comparing instances in which the phenomenon does occur, with instances in other respects similar in which it does not. These two methods may be respectively denominated the Method of Agreement and the Method of Difference.

* * *

We shall denote antecedents by the large letters of the alphabet, and the consequents corresponding to them by the small. Let A, then, be an agent or cause, and let the object of our inquiry be to ascertain what are the effects of this cause. If we can either find or produce the agent A in such varieties of circumstances that the different cases have no circumstance in common except A, then whatever effect we find to be produced in all our trials is indicated as the effect of A. Suppose, for example, that A is tried along with B and C, and that the effect is *a b c*; and suppose that A is next tried with D and E, but without B and C, and that the effect is *a d e*. Then we may reason thus: *b* and *c* are not effects of A, for they were not produced by it in the second experiment; nor are *d* and *e*, for they were not produced in the first. Whatever is really the effect of A must have been produced in both instances; now this condition is fulfilled by no circumstance except *a*. The phenomenon *a* cannot have been

the effect of B or C, since it was produced where they were not; nor of D or E, since it was produced where they were not. Therefore it is the effect of A.

For example, let the antecedent A be the contact of an alkaline substance and an oil. This combination being tried under several varieties of circumstances, resembling each other in nothing else, the results agree in the production of a greasy and deterative or saponaceous substance: it is therefore concluded that the combination of an oil and an alkali causes the production of a soap. It is thus we inquire, by the Method of Agreement, into the effect of a given cause.

In a similar manner we may inquire into the cause of a given effect. Let *a* be the effect. Here, as shown in the last chapter, we have only the resource of observation without experiment: we cannot take a phenomenon of which we know not the origin, and try to find its mode of production by producing it: if we succeeded in such a random trial it could only be by accident. But if we can observe *a* in two different combinations, *a b c* and *a d e*; and if we know, or can discover, that the antecedent circumstances in these cases respectively were A B C and A D E, we may conclude by a reasoning similar to that in the preceding example, that A is the antecedent connected with the consequent *a* by a law of causation. B and C, we may say, cannot be causes of *a*, since on its second occurrence they were not present; nor are D and E, for they were not present on its first occurrence. A, alone of the five circumstances, was found among the antecedents of *a* in both instances.

* * *

The mode of discovering and proving laws of nature, which we have now examined, proceeds on the following axiom. Whatever circumstances can be excluded, without prejudice to the phenomenon, or can be absent notwithstanding its presence, is not connected with it in the way of causation. The casual circumstance being thus eliminated, if only one remains, that one is the cause which we are in search of: if more than one, they either are, or contain among them, the cause; and so, *mutatis*

mutandis, of the effect. As this method proceeds by comparing different instances to ascertain in what they agree, I have termed it the Method of Agreement; and we may adopt as its regulating principle the following canon:—

FIRST CANON

If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon.

Quitting for the present the Method of Agreement, to which we shall almost immediately return, we proceed to a still more potent instrument of the investigation of nature, the Method of Difference.

§2. In the Method of Agreement, we endeavoured to obtain instances which agreed in the given circumstance but differed in every other: in the present method we require, on the contrary, two instances resembling one another in every other respect, but differing in the presence or absence of the phenomenon we wish to study. If our object be to discover the effects of an agent A, we must procure A in some set of ascertained circumstances, as A B C, and having noted the effects produced, compare them with the effect of the remaining circumstances B C, when A is absent. If the effect of A B C is *a b c*, and the effect of B C, *b c*, it is evident that the effect of A is *a*. So again, if we begin at the other end, and desire to investigate the cause of an effect *a*, we must select an instance, as *a b c*, in which the effect occurs, and in which the antecedents were A B C, and we must look out for another instance in which the remaining circumstances, *b c*, occur without *a*. If the antecedents, in that instance, are B C, we know that the cause of *a* must be A: either A alone, or A in conjunction with some of the other circumstances present.

It is scarcely necessary to give examples of a logical process to which we owe almost all the inductive conclusions we draw in early life. When a man is shot through the heart, it is by this method we know that it was the gunshot which killed him: for

he was in the fulness of life immediately before, all circumstances being the same, except the wound.

The axioms implied in this method are evidently the following. Whatever antecedent cannot be excluded without preventing the phenomenon, is the cause, or a condition of that phenomenon: Whatever consequent can be excluded, with no other difference in the antecedents than the absence of a particular one, is the effect of that one. Instead of comparing different instances of a phenomenon, to discover in what they agree, this method compares an instance of its occurrence with an instance of its non-occurrence, to discover in what they differ. The canon which is the regulating principle of the Method of Difference may be expressed as follows:—

SECOND CANON

If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.

§3. The two methods which we have now stated have many features of resemblance, but there are also many distinctions between them. Both are methods of *elimination*. This term (employed in the theory of equations to denote the process by which one after another of the elements of a question is excluded, and the solution made to depend on the relation between the remaining elements only) is well suited to express the operation, analogous to this, which has been understood since the time of Bacon to be the foundation of experimental inquiry, namely, the successive exclusion of the various circumstances which are found to accompany a phenomenon in a given instance, in order to ascertain what are those among them which can be absent consistently with the existence of the phenomenon. The Method of Agreement stands on the ground that whatever can be eliminated is not connected with the phenomenon by any law. The Method of Difference has for its foundation,

that whatever cannot be eliminated is connected with the phenomenon by a law.

* * *

It thus appears to be by the Method of Difference alone that we can ever, in the way of direct experience, arrive with certainty at causes. The Method of Agreement leads only to laws of phenomena, (as some writers call them, but improperly, since laws of causation are also laws of phenomena,) that is, to uniformities, which either are not laws of causation, or in which the question of causation must for the present remain undecided. The Method of Agreement is chiefly to be resorted to as a means of suggesting applications of the Method of Difference, (as in the last example the comparison of A B C, A D E, A F G, suggested that A was the antecedent on which to try the experiment whether it could produce *a*,) or as an inferior resource in case the Method of Difference is impracticable; which, as we before showed, generally arises from the impossibility of artificially producing the phenomena. And hence it is that the Method of Agreement, though applicable in principle to either case, is more emphatically the method of investigation on those subjects where artificial experimentation is impossible; because on those it is generally our only resource of a directly inductive nature; while, in the phenomena which we can produce at pleasure, the Method of Difference generally affords a more efficacious process, which will ascertain causes as well as mere laws.

§4. There are, however, many cases in which, though our power of producing the phenomenon is complete, the Method of Difference either cannot be made available at all, or not without a previous employment of the Method of Agreement. This occurs when the agency by which we can produce the phenomenon is not that of one single antecedent, but a combination of antecedents, which we have no power of separating from each other and exhibiting apart. For instance, suppose the subject of inquiry to be the cause of the double refraction of light. We can produce this phenomenon at pleasure by employing any one of the many substances which are known to refract light in that

peculiar manner. But if, taking one of those substances, as Iceland spar, for example, we wish to determine on which of the properties of Iceland spar this remarkable phenomenon depends, we can make no use for that purpose of the Method of Difference; for we cannot find another substance precisely resembling Iceland spar except in some one property. The only mode, therefore, of prosecuting this inquiry is that afforded by the Method of Agreement; by which, in fact, through a comparison of all the known substances which have the property of doubly refracting light, it was ascertained that they agree in the circumstance of being crystalline substances; and though the converse does not hold, though all crystalline substances have not the property of double refraction, it was concluded, with reason, that there is a real connection between these two properties; that either crystalline structure, or the cause which gives rise to that structure, is one of the conditions of double refraction.

Out of this employment of the Method of Agreement arises a peculiar modification of that method, which is sometimes of great avail in the investigation of nature. In cases similar to the above, in which it is not possible to obtain the precise pair of instances which our second canon requires—instances agreeing in every antecedent except A, or in every consequent except *a*—we may yet be able, by a double employment of the Method of Agreement, to discover in what the instances which contain A or *a* differ from those which do not.

If we compare various instances in which *a* occurs, and find that they all have in common the circumstance A, and (as far as can be observed) no other circumstance, the Method of Agreement, so far, bears testimony to a connection between A and *a*. In order to convert this evidence of connection into proof of causation by the direct Method of Difference, we ought to be able, in some one of these instances, as, for example, A B C, to leave out A, and observe whether by doing so *a* is prevented. Now supposing (what is often the case) that we are not able to try this decisive experiment, yet, provided we can by any means discover what would be its result if we could try it, the advan-

tage will be the same. Suppose, then, that as we previously examined a variety of instances in which *a* occurred, and found them to agree in containing A, so we now observe a variety of instances in which *a* does not occur, and find them agree in not containing A; which establishes, by the Method of Agreement, the same connection between the absence of A and the absence of *a*, which was before established between their presence. As, then, it had been shown that whenever A is present *a* is present, so it being now shown that when A is taken away *a* is removed along with it, we have by the one proposition A B C, *a b c*, by the other B C, *b c*, the positive and negative instances which the Method of Difference requires.

This method may be called the Indirect Method of Difference, or the Joint Method of Agreement and Difference, and consists in a double employment of the Method of Agreement, each proof being independent of the other, and corroborating it. But it is not equivalent to a proof by the direct Method of Difference. For the requisitions of the Method of Difference are not satisfied unless we can be quite sure either that the instances affirmative of *a* agree in no antecedent whatever but A, or that the instances negative of *a* agree in nothing but the negation of A. Now if it were possible, which it never is, to have this assurance, we should not need the joint method; for either of the two sets of instances separately would then be sufficient to prove causation. This indirect method, therefore, can only be regarded as a great extension and improvement of the Method of Agreement, but not as participating in the more cogent nature of the Method of Difference. The following may be stated as its canon:—

THIRD CANON

If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.

We shall presently see that the Joint Method of Agreement and Difference constitutes, in another respect not yet adverted to, an improvement upon the common Method of Agreement, namely, in being unaffected by a characteristic imperfection of that method, the nature of which still remains to be pointed out. But as we cannot enter into this exposition without introducing a new element of complexity into this long and intricate discussion, I shall postpone it to a subsequent chapter, and shall at once proceed to a statement of two other methods, which will complete the enumeration of the means which mankind possess for exploring the laws of nature by specific observation and experience.

§5. The first of these has been aptly denominated the Method of Residues. Its principle is very simple. Subducting from any given phenomenon all the portions which, by virtue of preceding inductions, can be assigned to known causes, the remainder will be the effect of the antecedents which had been overlooked, or of which the effect was as yet an unknown quantity.

Suppose, as before, that we have the antecedents A B C, followed by the consequents *a b c*, and that by previous inductions (founded, we will suppose, on the Method of Difference) we have ascertained the causes of some of these effects, or the effects of some of these causes; and are thence apprised that the effect of A is *a*, and that the effect of B is *b*. Subtracting the sum of these effects from the total phenomenon, there remains *c*, which now, without any fresh experiments, we may know to be the effect of C. This Method of Residues is in truth a peculiar modification of the Method of Difference. If the instance A B C, *a b c*, could have been compared with a single instance A B, *a b*, we should have proved C to be the cause of *c*, by the common process of the Method of Difference. In the present case, however, instead of a single instance A B, we have had to study separately the causes A and B, and to infer from the effects which they produce separately what effect they must produce in the case A B C where they act together. Of the two instances, therefore, which the Method of Difference requires,—the one positive, the other negative,—the negative one, or that in which the given

phenomenon is absent, is not the direct result of observation and experiment, but has been arrived at by deduction. As one of the forms of the Method of Difference, the Method of Residues partakes of its rigorous certainty, provided the previous inductions, those which gave the effects of A and B, were obtained by the same infallible method, and provided we are certain that C is the *only* antecedent to which the residual phenomenon *c* can be referred; the only agent of which we had not already calculated and subducted the effect. But as we can never be quite certain of this, the evidence derived from the Method of Residues is not complete unless we can obtain C artificially and try it separately, or unless its agency, when once suggested, can be accounted for, and proved deductively, from known laws.

Even with these reservations, the Method of Residues is one of the most important among our instruments of discovery. Of all the methods of investigating laws of nature, this is the most fertile in unexpected results: often informing us of sequences in which neither the cause nor the effect were sufficiently conspicuous to attract of themselves the attention of observers. The agent C may be an obscure circumstance, not likely to have been perceived unless sought for, nor likely to have been sought for until attention had been awakened by the insufficiency of the obvious causes to account for the whole of the effect. And *c* may be so disguised by its intermixture with *a* and *b*, that it would scarcely have presented itself spontaneously as a subject of separate study. Of these uses of the method we shall presently cite some remarkable examples. The canon of the Method of Residues is as follows:—

FOURTH CANON

Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.

§6. There remains a class of laws which it is impracticable to ascertain by any of the three methods which I have attempted to characterise, namely, the laws of those Permanent Causes, or

indestructible natural agents, which it is impossible either to exclude or to isolate; which we can neither hinder from being present, nor contrive that they shall be present alone. It would appear at first sight that we could by no means separate the effects of these agents from the effects of those other phenomena with which they cannot be prevented from co-existing.

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As another example, let us take the phenomenon Heat. Independently of all hypothesis as to the real nature of the agency so called, this fact is certain, that we are unable to exhaust any body of the whole of its heat. It is equally certain that no one ever perceived heat not emanating from a body. Being unable, then, to separate Body and Heat, we cannot effect such a variation of circumstances as the foregoing three methods require; we cannot ascertain, by those methods, what portion of the phenomena exhibited by any body is due to the heat contained in it. If we could observe a body with its heat, and the same body entirely divested of heat, the Method of Difference would show the effect due to the heat, apart from that due to the body. If we could observe heat under circumstances agreeing in nothing but heat, and therefore not characterised also by the presence of a body, we could ascertain the effects of heat, from an instance of heat with a body and an instance of heat without a body, by the Method of Agreement; or we could determine by the Method of Difference what effect was due to the body, when the remainder which was due to the heat would be given by the Method of Residues. But we can do none of these things; and without them the application of any of the three methods to the solution of this problem would be illusory. It would be idle, for instance, to attempt to ascertain the effect of heat by subtracting from the phenomena exhibited by a body all that is due to its other properties; for as we have never been able to observe any bodies without a portion of heat in them, effects due to that heat might form a part of the very results which we were affecting to subtract in order that the effect of heat might be shown by the residue.

If, therefore, there were no other methods of experimental investigation than these three, we should be unable to determine the effects due to heat as a cause. But we have still a resource. Though we cannot exclude an antecedent altogether, we may be able to produce, or nature may produce for us, some modification in it. By a modification is here meant a change in it, not amounting to its total removal. If some modification in the antecedent *A* is always followed by a change in the consequent *a*, the other consequents *b* and *c* remaining the same; or *vice versa*, if every change in *a* is found to have been preceded by some modification in *A*, none being observable in any of the other antecedents; we may safely conclude that *a* is, wholly or in part, an effect traceable to *A*, or at least in some way connected with it through causation. For example, in the case of heat, though we cannot expel it altogether from any body, we can modify it in quantity, we can increase or diminish it; and doing so, we find by the various methods of experimentation or observation already treated of, that such increase or diminution of heat is followed by expansion or contraction of the body. In this manner we arrive at the conclusion, otherwise unattainable by us, that one of the effects of heat is to enlarge the dimensions of bodies; or what is the same thing in other words, to widen the distances between their particles.

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That the oscillations of the pendulum are caused by the earth is proved by similar evidence. Those oscillations take place between equidistant points on the two sides of a line, which, being perpendicular to the earth, varies with every variation in the earth's position, either in space or relatively to the object. Speaking accurately, we only know by the method now characterised that all terrestrial bodies tend to the earth, and not to some unknown fixed point lying in the same direction. In every twenty-four hours, by the earth's rotation, the line drawn from the body at right angles to the earth coincides successively with all the radii of a circle, and in the course of six months the place of that circle varies by nearly two hundred millions of miles; yet

in all these changes of the earth's position, the line in which bodies tend to fall continues to be directed towards it: which proves that terrestrial gravity is directed to the earth, and not, as was once fancied by some, to a fixed point of space.

The method by which these results were obtained may be termed the Method of Concomitant Variations: it is regulated by the following canon:—

FIFTH CANON

Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.

The last clause is subjoined because it by no means follows, when two phenomena accompany each other in their variations, that the one is cause and the other effect. The same thing may, and indeed must happen, supposing them to be two different effects of a common cause: and by this method alone it would never be possible to ascertain which of the suppositions is the true one. The only way to solve the doubt would be that which we have so often adverted to, viz. by endeavouring to ascertain whether we can produce the one set of variations by means of the other. In the case of heat, for example, by increasing the temperature of a body we increase its bulk, but by increasing its bulk we do not increase its temperature; on the contrary, (as in the rarefaction of air under the receiver of an air-pump,) we generally diminish it: therefore heat is not an effect, but a cause, of increase of bulk. If we cannot ourselves produce the variations, we must endeavour, though it is an attempt which is seldom successful, to find them produced by nature in some case in which the pre-existing circumstances are perfectly known to us.

It is scarcely necessary to say, that in order to ascertain the uniform concomitants of variations in the effect with variations in the cause, the same precautions must be used as in any other case of the determination of an invariable sequence. We must endeavour to retain all the other antecedents unchanged, while

that particular one is subjected to the requisite series of variations; or, in other words, that we may be warranted in inferring causation from concomitance of variations, the concomitance itself must be proved by the Method of Difference.

* * *

Although the most striking applications of the Method of Concomitant Variations take place in the cases in which the Method of Difference, strictly so called, is impossible, its use is not confined to those cases; it may often usefully follow after the Method of Difference, to give additional precision to a solution which that has found. When by the Method of Difference it has first been ascertained that a certain object produces a certain effect, the Method of Concomitant Variations may be usefully called in to determine according to what law the quantity or the different relations of the effect follow those of the cause.

§7. The case in which this method admits of the most extensive employment is that in which the variations of the cause are variations of quantity. Of such variations we may in general affirm with safety that they will be attended not only with variations, but with similar variations of the effect: the proposition, that more of the cause is followed by more of the effect, being a corollary from the principle of the Composition of Causes, which, as we have seen, is the general rule of causation; cases of the opposite description, in which causes change their properties on being conjoined with one another, being, on the contrary, special and exceptional. Suppose, then, that when A changes in quantity, *a* also changes in quantity, and in such a manner that we can trace the numerical relation which the changes of the one bear to such changes of the other as take place within our limits of observation. We may then, with certain precautions, safely conclude that the same numerical relation will hold beyond those limits. If, for instance, we find that when A is double, *a* is double; that when A is treble or quadruple, *a* is treble or quadruple; we may conclude that if A were

a half or a third, *a* would be a half or a third; and finally, that if A were annihilated, *a* would be annihilated; and that *a* is wholly the effect of A, or wholly the effect of the same cause with A. And so with any other numerical relation according to which A and *a* would vanish simultaneously; as, for instance, if *a* were proportional to the square of A. If, on the other hand, *a* is not wholly the effect of A, but yet varies when A varies, it is probably a mathematical function not of A alone, but of A and something else; its changes, for example, may be such as would occur if part of it remained constant, or varied on some other principle, and the remainder varied in some numerical relation to the variations of A. In that case, when A diminishes, *a* will be seen to approach not towards zero, but towards some other limit; and when the series of variations is such as to indicate what that limit is, if constant, or the law of its variation if variable, the limit will exactly measure how much of *a* is the effect of some other and independent cause, and the remainder will be the effect of A (or of the cause of A).

These conclusions, however, must not be drawn without certain precautions. In the first place, the possibility of drawing them at all manifestly supposes that we are acquainted not only with the variations, but with the absolute quantities both of A and *a*. If we do not know the total quantities, we cannot, of course, determine the real numerical relation according to which those quantities vary. It is therefore an error to conclude, as some have concluded, that because increase of heat expands bodies, that is, increases the distance between their particles, therefore the distance is wholly the effect of heat, and that if we could entirely exhaust the body of its heat, the particles would be in complete contact. This is no more than a guess, and of the most hazardous sort, not a legitimate induction; for since we neither know how much heat there is in any body, nor what is the real distance between any two of its particles, we cannot judge whether the contraction of the distance does or does not follow the diminution of the quantity of heat according to such a numerical relation that the two quantities would vanish simultaneously.

* * *

There is also another characteristic uncertainty affecting the inference that the law of variation, which the quantities observe within our limits of observation, will hold beyond those limits. There is, of course, in the first instance, the possibility that beyond the limits, and in circumstances therefore of which we have no direct experience, some counteracting cause might develop itself; either a new agent, or a new property of the agents concerned, which lies dormant in the circumstances we are able to observe. This is an element of uncertainty which enters largely into all our predictions of effects; but it is not peculiarly applicable to the Method of Concomitant Variations. The uncertainty, however, of which I am about to speak is characteristic of that method, especially in the cases in which the extreme limits of our observation are very narrow in comparison with the possible variations in the quantities of the phenomena. Any one who has the slightest acquaintance with mathematics is aware that very different laws of variation may produce numerical results which differ but slightly from one another within narrow limits; and it is often only when the absolute amounts of variation are considerable that the difference between the results given by one law and by another becomes appreciable. When, therefore, such variations in the quantity of the antecedents as we have the means of observing are small in comparison with the total quantities, there is much danger lest we should mistake the numerical law, and be led to miscalculate the variations which would take place beyond the limits; a miscalculation which would vitiate any conclusion respecting the dependence of the effect upon the cause, that could be founded on those variations. Examples are not wanting of such mistakes. . . .

In this uncertainty, the conclusion we may draw from the concomitant variations of *a* and *A*, to the existence of an invariable and exclusive connection between them, or to the permanency of the same numerical relation between their variations when the quantities are much greater or smaller than

those which we have had the means of observing, cannot be considered to rest on a complete induction. All that in such a case can be regarded as proved on the subject of causation is, that there is some connection between the two phenomena; that *A*, or something which can influence *A*, must be *one* of the causes which collectively determine *a*. We may, however, feel assured that the relation which we have observed to exist between the variations of *A* and *a*, will hold true in all cases which fall between the same extreme limits; that is, wherever the utmost increase or diminution in which the result has been found by observation to coincide with the law, is not exceeded.

The four methods which it has now been attempted to describe are the only possible modes of experimental inquiry—of direct induction *à posteriori*, as distinguished from deduction: at least, I know not, nor am able to imagine, any others. And even of these, the Method of Residues, as we have seen, is not independent of deduction; though, as it also requires specific experience, it may, without impropriety, be included among methods of direct observation and experiment.

These, then, with such assistance as can be obtained from Deduction, compose the available resources of the human mind for ascertaining the laws of the succession of phenomena.

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