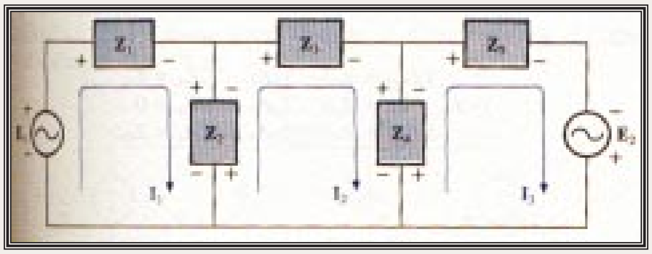
CIRCUIT ANALYSIS II

Chapter 2

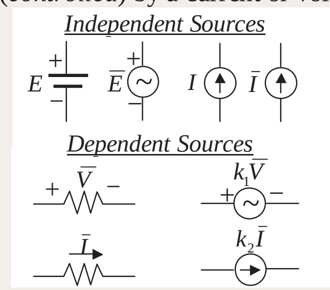
AC Circuits Analysis Techniques

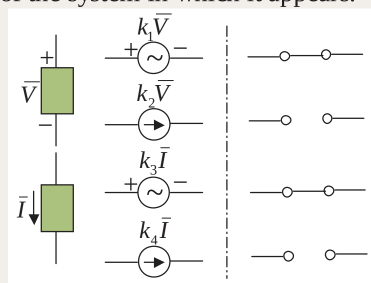


CONTENTS

2.1 Independent Versus Dependent Sources

- ***Independent source***: The magnitude of source is independent of the network to which it is applied and it displays its terminal characteristics even if completely isolated.
- ***Dependent source***: The magnitude of a dependent source is determined (*controlled*) by a current or voltage of the system in which it appears.

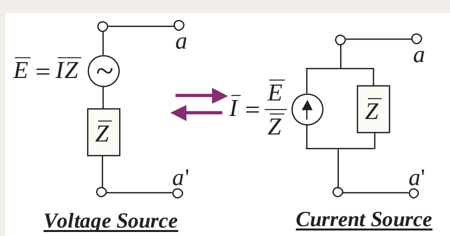




Note that the magnitude of current sources or voltage sources can be controlled by a voltage and a current, respectively. Isolation such as V or I =0 will result in the short-circuit or open-circuit equivalent.

2.2 Source Conversions

- Sometimes, it might be necessary to convert a current source to voltage source and vice-versa.
- This can be done in a similar manner as that of dc circuits.

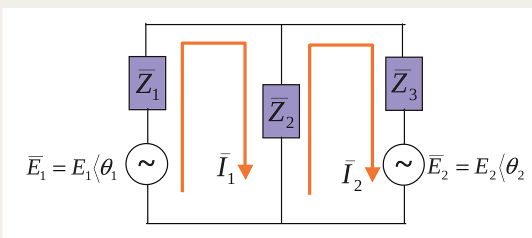


2.3 Mesh Analysis Method

- ❖ The mesh analysis uses the format approach, introduced in Circuit analysis I (dc circuits).
- ❖ The only change as compared to its appearance in dc circuits is to replace the word *resistor* by the word *impedance*. This and the use of phasors instead of real numbers will be the only major changes in applying this method to ac circuits.
- ❖ The steps for applying this method are repeated here with changes for its use in ac circuits.

STEPS:

1. Assign a loop current to each independent closed loop in a *clockwise* direction.



2. The number of required equations is equal to the number of chosen independent closed loops. Column 1 of each equation is formed by simply summing the impedance values of those impedances through which the loop current of interest passes and multiplying the result by that loop current.

3. We must now consider the mutual terms which are always subtracted from the terms in the first column. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. Each mutual term is the product of the mutual impedance and the other loop current passing through the same element.

4. The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. A negative sign is assigned to those potentials for which the reverse is true.

$$\begin{aligned}\bar{I}_1(\bar{Z}_1+\bar{Z}_2)-\bar{I}_2\bar{Z}_2&=\bar{E}_1\\ \bar{I}_2(\bar{Z}_2+\bar{Z}_3)-\bar{I}_1\bar{Z}_2&=-\bar{E}_2\end{aligned} \quad\Leftrightarrow\quad \begin{aligned}\bar{I}_1(\bar{Z}_1+\bar{Z}_2)-\bar{I}_2\bar{Z}_2\quad&=\bar{E}_1\\ -\bar{I}_1\bar{Z}_2\quad+\bar{I}_2(\bar{Z}_2+\bar{Z}_3)&=-\bar{E}_2\end{aligned}$$

5. Solve the resulting simultaneous equations for the desired loop currents. Using the determinants, we can obtain:

$$\bar{I}_1=\frac{\begin{vmatrix}\bar{E}_1 & -\bar{Z}_2\\ -\bar{E}_2 & \bar{Z}_2+\bar{Z}_3\end{vmatrix}}{\begin{vmatrix}\bar{Z}_1+\bar{Z}_2 & -\bar{Z}_2\\ -\bar{Z}_2 & \bar{Z}_2+\bar{Z}_3\end{vmatrix}}=\frac{\bar{E}_1(\bar{Z}_2+\bar{Z}_3)-\bar{E}_2(\bar{Z}_2)}{(\bar{Z}_1+\bar{Z}_2)(\bar{Z}_2+\bar{Z}_3)-(\bar{Z}_2)^2}=\frac{(\bar{E}_1-\bar{E}_2)\bar{Z}_2+\bar{E}_1\bar{Z}_3}{\bar{Z}_1\bar{Z}_2+\bar{Z}_1\bar{Z}_3+\bar{Z}_2\bar{Z}_3}$$

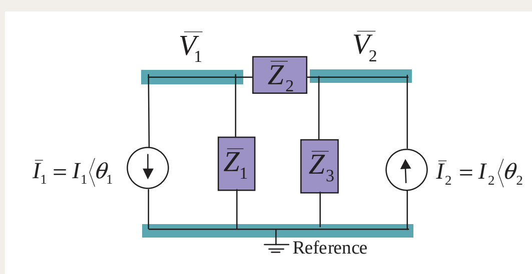
$$\bar{I}_2=\frac{\begin{vmatrix}\bar{Z}_1+\bar{Z}_2 & \bar{E}_1\\ -\bar{Z}_2 & -\bar{E}_2\end{vmatrix}}{\begin{vmatrix}\bar{Z}_1+\bar{Z}_2 & -\bar{Z}_2\\ -\bar{Z}_2 & \bar{Z}_2+\bar{Z}_3\end{vmatrix}}=\frac{-\bar{E}_2(\bar{Z}_1+\bar{Z}_2)+\bar{E}_1(\bar{Z}_2)}{(\bar{Z}_1+\bar{Z}_2)(\bar{Z}_2+\bar{Z}_3)-(\bar{Z}_2)^2}=\frac{(\bar{E}_1-\bar{E}_2)\bar{Z}_2-\bar{E}_2\bar{Z}_1}{\bar{Z}_1\bar{Z}_2+\bar{Z}_1\bar{Z}_3+\bar{Z}_2\bar{Z}_3}$$

2.4 Nodal Analysis Method

- Before examining the application of this method to ac circuits, the students should review the section on nodal analysis in dc circuits, since we shall repeat only the final conclusions as they apply to ac circuits.
- Recall that these conclusions made the writing of the nodal equations quite direct and in a form convenient for the use of determinants.
- For sinusoidal ac networks, the procedure is the following:

STEPS

1. Choose a reference node and assign a subscripted voltage label to the *(N - 1)* remaining independent nodes of the network.



2. The number of equations required for a complete solution is equal to the number of subscripted voltages (*N* - *1*). Column 1 of each equation is formed by summing the admittances tied to the node of interest and multiplying the result by that subscripted nodal voltage.

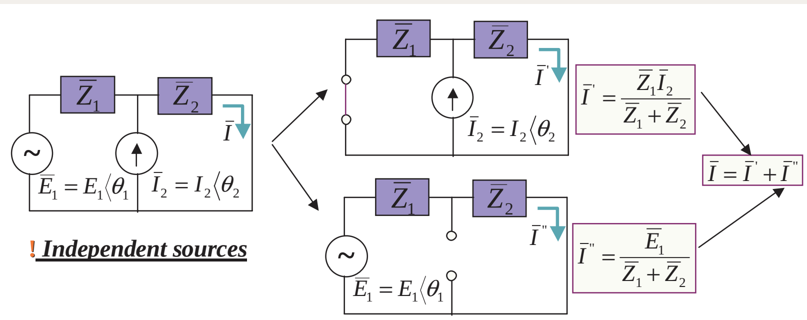
$$\begin{aligned}\bar V_1(\bar Y_1+\bar Y_2)-\bar V_2\bar Y_2&=-\bar I_1\\ \bar V_2(\bar Y_2+\bar Y_3)-\bar V_1\bar Y_2&=\bar I_2\end{aligned}\quad\Longleftrightarrow\quad\begin{aligned}\bar V_1(\bar Y_1+\bar Y_2)-\bar V_2\bar Y_2&=-\bar I_1\\ -\bar V_1\bar Y_2\qquad+\bar V_2(\bar Y_2+\bar Y_3)&=\bar I_2\end{aligned}$$

3. The mutual terms are always subtracted from the terms of the first column. It is possible to have more than one mutual term if the nodal voltage of interest has an element in common with more than one other nodal voltage. Each mutual term is the product of the mutual admittance and the other nodal voltage tied to that admittance.
4. The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node, and a negative sign if it draws current from the node.
5. Solve resulting simultaneous equations for the desired nodal voltages.

$$\bar V_1=\frac{\begin{vmatrix}-\bar I_1 & -\bar Y_2\\ \bar I_2 & \bar Y_2+\bar Y_3\end{vmatrix}}{\begin{vmatrix}\bar Y_1+\bar Y_2 & -\bar Y_2\\ -\bar Y_2 & \bar Y_2+\bar Y_3\end{vmatrix}}=\frac{-\bar I_1(\bar Y_2+\bar Y_3)+\bar I_2(\bar Y_2)}{(\bar Y_1+\bar Y_2)(\bar Y_2+\bar Y_3)-(\bar Y_2)^2}=\frac{(\bar I_2-\bar I_1)\bar Y_2-\bar I_1\bar Y_3}{\bar Y_1\bar Y_2+\bar Y_1\bar Y_3+\bar Y_2\bar Y_3}$$

2.5 Superposition Theorem

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.



! Dependent sources

- For dependent source for which the controlling variable is not determined by the network to which the superposition theorem is to be applied, the application of the theorem is basically the same as for the independent sources. The solution obtained will simply be in terms of the controlling variables.

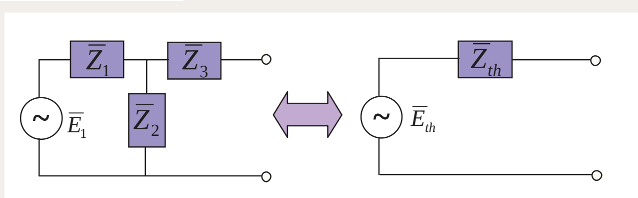
- For dependent sources in which the controlling variable is determined by the network to which the theorem is to be applied, the dependent source cannot be set to zero unless the controlling variable is also zero.

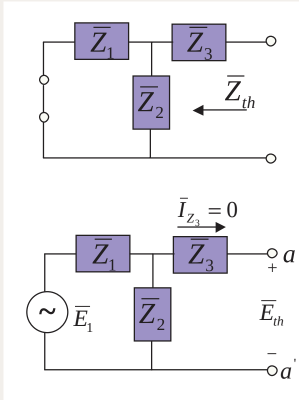
- *It should be emphasized that dependent sources are not sources of energy in the sense that if all independent sources are removed from the system, all currents and voltages must be zero.*

2.6 Thevenin's Theorem

Any two-terminal linear ac network can be replaced by an equivalent circuit consisting of a voltage source and an impedance in series.

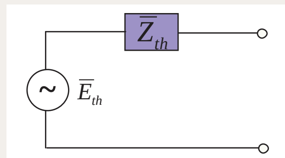
Independent Sources





$$\bar{Z}_{th} = \bar{Z}_3 + \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$\bar{E}_{th} = \frac{\bar{Z}_2 \bar{E}_1}{\bar{Z}_1 + \bar{Z}_2}$$ (Voltage divider rule)



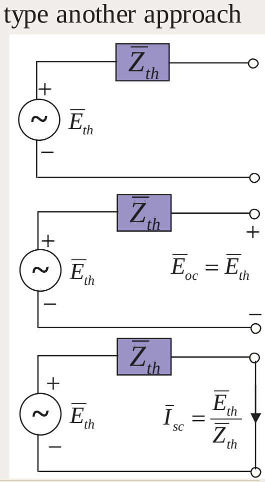
Dependent Sources

For dependent sources with controlling variable not in the network under investigation, the same procedure as indicated above (Approach 1) can be applied. However, for dependent sources of the other type another approach must be employed.

Approach 2:

If the open circuit terminal voltage of a portion of a network can be determined along with the short-circuit current between the same two terminals the Thevenin equivalent circuit can be effectively known as shown in the figure.

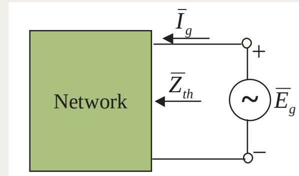
This method can be applied to both Independent and dependent sources.
The drawback is that, in general, the current I_{sc} is difficult to obtain since all sources are present.



Approach 3:

The Thevenin voltage is found as in the two previous approaches.

However, the Thevenin impedance is obtained by applying a source of voltage to the terminals of interest and determining the source current.



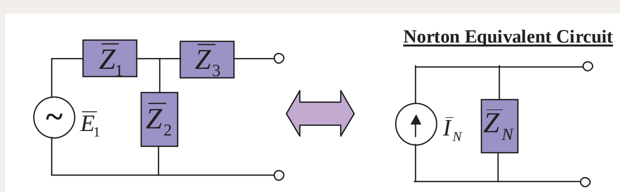
Determining Z_{th}.

For this method, the source voltage of the original network is set to zero. The Thevenin impedance is then determined by the following equation:

$$\bar{Z}_{th} = \frac{\bar{E}_g}{\bar{I}_g} \qquad \bar{E}_{th} = \bar{E}_{oc}$$

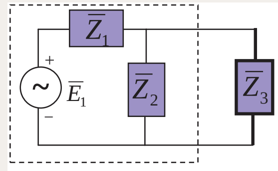
2.7 Norton's Theorem

Any two-terminal linear ac network can be replaced by an equivalent circuit consisting of a current source and a parallel impedance.

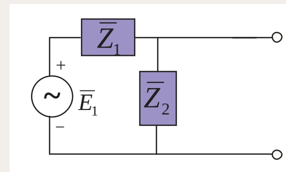


Independent Sources

Find the Norton equivalent circuit for the network external to Z_3 impedance.

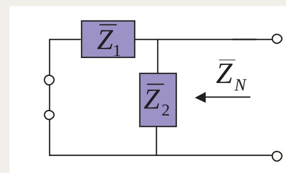


- Remove the portion of the network across which the Norton equivalent circuit is to be found.
- Mark (O, •, and so on) the terminals of the remaining two-terminal network.



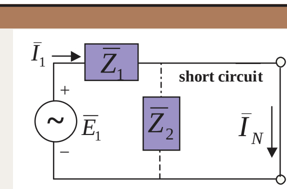
- Calculate Z_N by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two market terminals.

$$\bar{Z}_N = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$



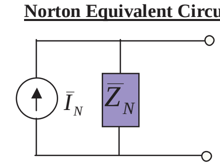
- Calculate I_N by first replacing all voltage and current sources and then finding the short-circuit current between the marked terminals.

$$\bar{I}_N = \bar{I}_1 = \frac{\bar{E}_1}{\bar{Z}_1}$$



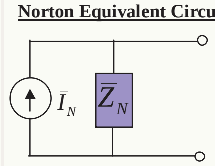
- Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals and the Norton equivalent circuit.

Norton Equivalent Circuit

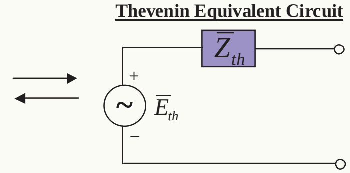


Note:
There is an analogy between Norton equivalent circuit and Thevenin equivalent circuit by applying the source (independent or dependent) transformation.

Norton Equivalent Circuit



Thevenin Equivalent Circuit

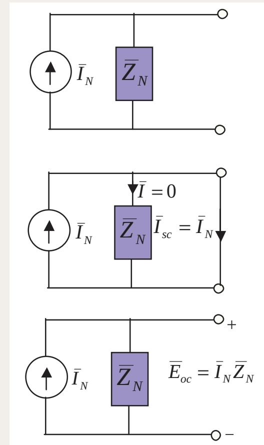


$$\bar{I}_N = \frac{\bar{E}_{th}}{\bar{Z}_{th}} \qquad \bar{Z}_N = \bar{Z}_{th}$$

$$\bar{E}_{th} = \bar{I}_N \bar{Z}_{th} \qquad \bar{Z}_N = \bar{Z}_{th}$$

Dependent Sources

For dependent sources with controlling variable not in the network under investigation, the same procedure as indicated above (Approach 1) can be applied. However, for dependent sources of the other type one of the following procedures must be employed.



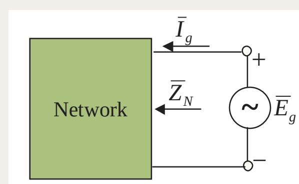
Approach 2:

If the open circuit terminal voltage of a portion of a network can be determined along with the short-circuit current between the same two terminals the Norton equivalent circuit can be effectively known as shown in the figure.

This method can be applied to both Independent and dependent sources (of any type).

Approach 3:

The Norton current is found as in the two previous approaches. $\bar{I}_N = \bar{I}_{sc}$



Determining Z_N.

However, the Norton impedance is obtained by applying a source of voltage to the terminals of interest and determining the resulting current I_g.

All sources (dependent + independent not controlled by a variable in the network of interest) of the original network are set to zero. The Norton impedance is then determined by the following equation:

$$\bar{Z}_N = \frac{\bar{E}_g}{\bar{I}_g}$$

The Norton current is still found by the short-circuit current.