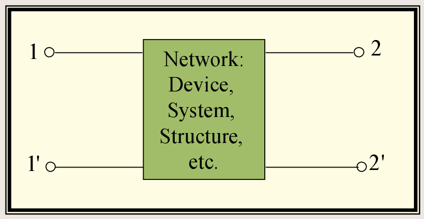
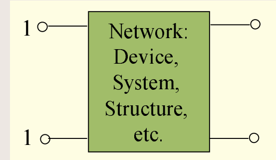
CIRCUIT ANALYSIS II
Chapter 3
General Tow-Port Networks

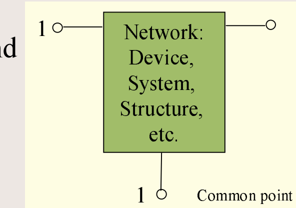


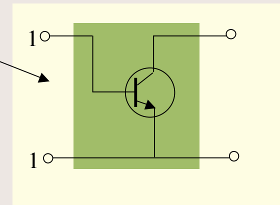
CONTENTS

3.1 Introduction

- Most of devices, systems and structures can be modeled with a 2-port network models.
- These models are employed in the analysis and synthesis of combined and enlarged systems.
- It is possible to establish a network that displays the same terminal characteristics as those of the original system, device and so on.
- For instance the transistor device shown in the figure can be modeled by a 2-port network.
- Methods such as mesh and nodal analysis can then be applied to determine any unknown quantities.



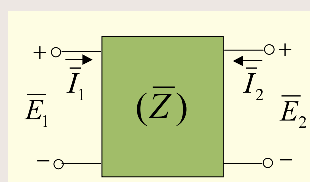




3.2 Impedance (z) Parameters

- For most situations, if any two variables of the network are known the remaining two variables can be determined.

$$\overline{E}_1 = \overline{z}_{11}\vec{I}_1 + \overline{z}_{12}\overline{I}_2 \qquad \overline{E}_2 = \overline{z}_{21}\vec{I}_1 + \overline{z}_{22}\overline{I}_2$$



All impedances are measured in Ohm.

- To model the system, each impedance parameter must be determined.
- They are determined by setting a particular current to zero (open-circuit).

$\overline{Z}_{11}$

$$\overline{I}_2 = 0 \Rightarrow \overline{E}_1 = \overline{z}_{11}\overline{I}_1 \Rightarrow \overline{z}_{11} = \left.\frac{\overline{E}_1}{\overline{I}_1}\right|_{\overline{I}_2=0}$$

Open-circuit, input-impedance parameter

$\overline{Z}_{12}$

$$\overline{I}_1 = 0 \Rightarrow \overline{E}_1 = \overline{z}_{12}\overline{I}_2 \Rightarrow \overline{z}_{12} = \left.\frac{\overline{E}_1}{\overline{I}_2}\right|_{\overline{I}_1=0}$$

Open-circuit, reverse-transfer impedance parameter

$\overline{Z}_{21}$

$$\overline{I}_2 = 0 \Rightarrow \overline{E}_2 = \overline{z}_{21}\overline{I}_1 \Rightarrow \overline{z}_{21} = \left.\frac{\overline{E}_2}{\overline{I}_1}\right|_{\overline{I}_2=0}$$

Open-circuit, forward-transfer impedance parameter

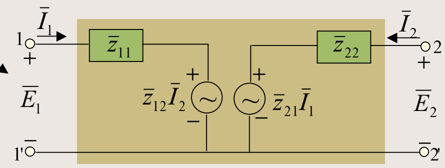
$\overline{Z}_{22}$

$$\overline{I}_1 = 0 \Rightarrow \overline{E}_2 = \overline{z}_{22}\overline{I}_2 \Rightarrow \overline{z}_{22} = \left.\frac{\overline{E}_2}{\overline{I}_2}\right|_{\overline{I}_1=0}$$

Open-circuit, output-impedance parameter

Two-Port z-Parameter Equivalent Network

From the previous set of equation we can obtain the following equivalent circuit.



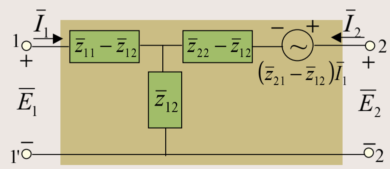
Applying Kirchhoff's voltage law :

$\bar{E}_1 = \bar{z}_{11}\bar{I}_1 + \bar{z}_{12}\bar{I}_2$ and

$\bar{E}_2 = \bar{z}_{22}\bar{I}_2 + \bar{z}_{21}\bar{I}_1$

These equations are the same as the previous ones which proves that the circuit is equivalent to previous two-port network.

Now from the next equivalent circuit, it is also possible to find the same set of equation which also proves that this circuit is equivalent to the above one.



$\bar{E}_1 - \bar{I}_1(\bar{z}_{11} - \bar{z}_{12}) - \bar{z}_{12}(\bar{I}_1 + \bar{I}_2) = 0$ and

$\bar{E}_2 - \bar{I}_1(\bar{z}_{21} - \bar{z}_{12}) - \bar{I}_2(\bar{z}_{22} - \bar{z}_{12}) - \bar{z}_{12}(\bar{I}_1 + \bar{I}_2) = 0$

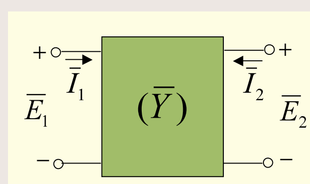
⇒

$\bar{E}_1 = \bar{z}_{11}\bar{I}_1 + \bar{z}_{12}\bar{I}_2$ and

$\bar{E}_2 = \bar{z}_{21}\bar{I}_1 + \bar{z}_{22}\bar{I}_2$

3.3 Admittance (y) Parameters

- Similarly to impedance parameter we can also define admittance parameter as follows.



$\bar{I}_1 = \bar{y}_{11}\bar{E}_1 + \bar{y}_1\ \bar{E}$ $\quad$ $\bar{I}_2 = \bar{y}_{21}\bar{E}_1 + \bar{y}_{22}\bar{E}_2$

All admittances are measured in Siemens.

- To model the system, each admittance parameter must be determined.
- They are determined by setting a particular voltage to zero (short-circuit).

$\bar{Y}_{11}$

$$\bar{y}_{11} = \left.\frac{\bar{I}_1}{\bar{E}_1}\right|_{\bar{E}_2=0}$$

Short-circuit, input-admittance parameter

$\bar{Y}_{12}$

$$\bar{y}_{12} = \left.\frac{\bar{I}_1}{\bar{E}_2}\right|_{\bar{E}_1=0}$$

Short-circuit, reverse-transfer admittance parameter

$\bar{Y}_{21}$

$$\bar{y}_{21} = \left.\frac{\bar{I}_2}{\bar{E}_1}\right|_{\bar{E}_2=0}$$

Short-circuit, forward-transfer admittance parameter

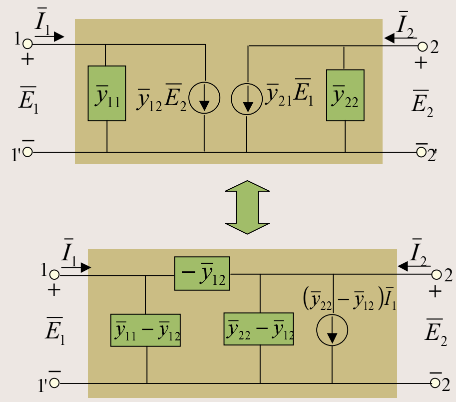
$\bar{Y}_{22}$

$$\bar{y}_{22} = \left.\frac{\bar{I}_2}{\bar{E}_2}\right|_{\bar{E}_1=0}$$

Short-circuit, output-admittance parameter

Two-Port y-parameter Equivalent Network

Following a similar to that for z-parameters, we can find the y-equivalent network as follows.



Applying Kirchhoff's current law can prove that the circuits are equivalent and give the same set of equations.

3.4 Hybrid (h) Parameters

The hybrid (h) parameters are employed extensively in the analysis of transistor networks.

The term hybrid is derived from the fact that the parameters have a mixture of units (Ohm, Siemens) and the defining equations have a mixture of voltage and current on one side.

$$\bar{E}_1 = \bar{h}_{11}\bar{I}_1 + \bar{h}_{12}\bar{E}_2 \qquad \bar{I}_2 = \bar{h}_{21}\bar{I}_1 + \bar{h}_{22}\bar{E}_2$$

$\bar{h}_{11}$

$$\bar{h}_{11} = \left.\frac{\bar{E}_1}{\bar{I}_1}\right|_{\bar{E}_2=0} \quad (\Omega)$$

Short-circuit, input-impedance parameter

$\bar{h}_{12}$

$$\bar{h}_{12} = \left.\frac{\bar{E}_1}{\bar{E}_2}\right|_{\bar{I}_2=0}$$

Open-circuit, reverse-transfer voltage ratio parameter

$\bar{h}_{21}$

$$\bar{h}_{21} = \left.\frac{\bar{I}_2}{\bar{I}_1}\right|_{\bar{E}_2=0}$$

Short-circuit, forward-transfer current ratio parameter

$\bar{h}_{22}$

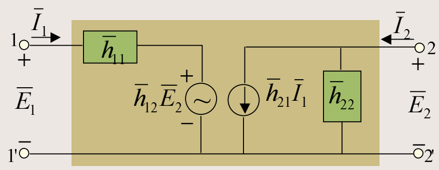
$$\bar{h}_{22} = \left.\frac{\bar{I}_2}{\bar{E}_2}\right|_{\bar{I}_1=0}$$

Open-circuit, output-admittance parameter

In many applications the subscript notation is reduced to the following:

$$\bar{h}_{11} = \bar{h}_i \qquad \bar{h}_{12} = \bar{h}_r \qquad \bar{h}_{21} = \bar{h}_f \qquad \bar{h}_{22} = \bar{h}_o$$

The two port hybrid-parameter equivalent circuit can be obtained as follows:



Note

- *The input circuit has a voltage-controlled voltage source whose controlling voltage is the output terminal voltage.*
- *The output circuit has a current-controlled current source whose controlling current is the current of the input circuit.*

3.5 Conversion Between Parameters

$$\left.\begin{aligned}\bar{I}_1 &= \bar{y}_{11}\bar{E}_1 + \bar{y}_{12}\bar{E}_2\\ \bar{I}_2 &= \bar{y}_{21}\bar{E}_1 + \bar{y}_{22}\bar{E}_2\end{aligned}\right\} \Rightarrow \bar{E}_1 = \frac{\begin{vmatrix}\bar{I}_1 & \bar{y}_{12}\\ \bar{I}_2 & \bar{y}_{22}\end{vmatrix}}{\begin{vmatrix}\bar{y}_{11} & \bar{y}_{12}\\ \bar{y}_{21} & \bar{y}_{22}\end{vmatrix}} = \frac{\bar{y}_{22}\bar{I}_1 - \bar{y}_{12}\bar{I}_2}{\bar{y}_{11}\bar{y}_{22} - \bar{y}_{12}\bar{y}_{21}} = \frac{\bar{y}_{22}}{\Delta_y}\bar{I}_1 - \frac{\bar{y}_{12}}{\Delta_y}\bar{I}_2$$

$$\Delta_y = \bar{y}_{11}\bar{y}_{22} - \bar{y}_{12}\bar{y}_{21}$$

$$\bar{E}_1 = \frac{\bar{y}_{22}}{\Delta_y}\bar{I}_1 - \frac{\bar{y}_{12}}{\Delta_y}\bar{I}_2 = \bar{z}_{11}\bar{I}_1 + \bar{z}_{12}\bar{I}_2$$

$$\bar{z}_{11} = \frac{\bar{y}_{22}}{\Delta_y} \qquad \bar{z}_{12} = -\frac{\bar{y}_{12}}{\Delta_y}$$

Similarly we can also obtain: $\bar{z}_{21} = -\dfrac{\bar{y}_{21}}{\Delta_y} \qquad \bar{z}_{22} = \dfrac{\bar{y}_{11}}{\Delta_y}$

$$\left.\begin{aligned}\bar{E}_1 &= \bar{h}_{11}\bar{I}_1 + \bar{h}_{12}\bar{E}_2 \\ \bar{I}_2 &= \bar{h}_{21}\bar{I}_1 + \bar{h}_{22}\bar{E}_2\end{aligned}\right\} \Rightarrow \bar{I}_1 = \frac{\begin{vmatrix}\bar{E}_1 & \bar{h}_{12} \\ \bar{I}_2 & \bar{h}_{22}\end{vmatrix}}{\begin{vmatrix}\bar{h}_{11} & \bar{h}_{12} \\ \bar{h}_{21} & \bar{h}_{22}\end{vmatrix}} = \frac{\bar{h}_{22}\bar{E}_1 - \bar{h}_{12}\bar{I}_2}{\bar{h}_{11}\bar{h}_{22} - \bar{h}_{12}\bar{h}_{21}} = \frac{\bar{h}_{22}}{\Delta_h}\bar{E}_1 - \frac{\bar{h}_{12}}{\Delta_h}\bar{I}_2$$

$$\Delta_h = \bar{h}_{11}\bar{h}_{22} - \bar{h}_{12}\bar{h}_{21}$$

$$\bar{E}_1 = \frac{\Delta_h}{\bar{h}_{22}}\bar{I}_1 + \frac{\bar{h}_{12}}{\bar{h}_{22}}\bar{I}_2$$

$$\bar{z}_{11} = \frac{\Delta_h}{\bar{h}_{22}} \qquad \bar{z}_{12} = \frac{\bar{h}_{12}}{\bar{h}_{22}}$$

Similarly we can also obtain: $\bar{z}_{21} = -\frac{\bar{h}_{12}}{\bar{h}_{22}} \qquad \bar{z}_{22} = \frac{1}{\bar{h}_{22}}$

Conversions between z, y, and h parameters

TO \ FROM	$\bar{z}$	$\bar{y}$	$\bar{h}$
$\bar{z}$	$\begin{matrix}\bar{z}_{11} & \bar{z}_{12} \\ \bar{z}_{21} & \bar{z}_{22}\end{matrix}$	$\begin{matrix}\frac{\bar{y}_{22}}{\Delta_y} & \frac{-\bar{y}_{12}}{\Delta_y} \\ \frac{-\bar{y}_{21}}{\Delta_y} & \frac{\bar{y}_{11}}{\Delta_y}\end{matrix}$	$\begin{matrix}\frac{\Delta_h}{\bar{h}_{22}} & \frac{\bar{h}_{12}}{\bar{h}_{22}} \\ -\frac{\bar{h}_{12}}{\bar{h}_{22}} & \frac{1}{\bar{h}_{22}}\end{matrix}$
$\bar{y}$	$\begin{matrix}\frac{\bar{z}_{22}}{\Delta_z} & \frac{-\bar{z}_{12}}{\Delta_z} \\ \frac{-\bar{z}_{21}}{\Delta_z} & \frac{\bar{z}_{11}}{\Delta_z}\end{matrix}$	$\begin{matrix}\bar{y}_{11} & \bar{y}_{12} \\ \bar{y}_{21} & \bar{y}_{22}\end{matrix}$	$\begin{matrix}\frac{1}{\bar{h}_{11}} & \frac{-\bar{h}_{12}}{\bar{h}_{11}} \\ \frac{\bar{h}_{21}}{\bar{h}_{11}} & \frac{\Delta_h}{\bar{h}_{11}}\end{matrix}$
$\bar{h}$	$\begin{matrix}\frac{\Delta_z}{\bar{z}_{22}} & \frac{\bar{z}_{12}}{\bar{z}_{22}} \\ -\frac{\bar{z}_{12}}{\bar{z}_{22}} & \frac{1}{\bar{z}_{22}}\end{matrix}$	$\begin{matrix}\frac{1}{\bar{y}_{22}} & \frac{-\bar{y}_{12}}{\bar{y}_{11}} \\ \frac{\bar{y}_{21}}{\bar{y}_{11}} & \frac{\Delta_y}{\bar{y}_{11}}\end{matrix}$	$\begin{matrix}\bar{h}_{11} & \bar{h}_{12} \\ \bar{h}_{21} & \bar{h}_{22}\end{matrix}$