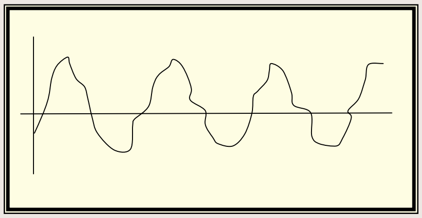
CIRCUIT ANALYSIS II

Chapter 4

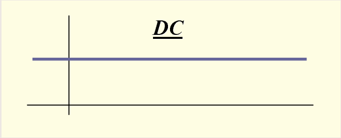
Non-Sinusoidal Circuits



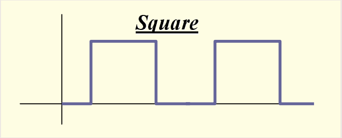
CONTENTS

4.1 Introduction

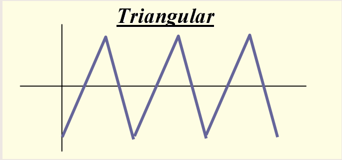
- Any waveform that differs from the basic description of sinusoidal waveform is referred to as ***nonsinusoidal***.
- DC and square-wave voltage or current are most obvious examples.



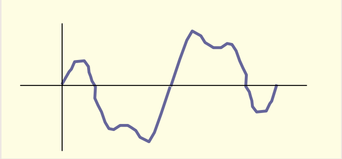
DC



Square



Triangular



4.2 Fourier Series

- Any *periodic* waveform can be represented by the sum of a dc component, cosine, and sine terms.
- This is called ***Fourier Series*** decomposition or representation. (Baron Jean Fourier 1826).

$$f(x)=\overset{\substack{\text{dc or}\\ \text{average value}\\ \downarrow}}{A_o}+\overbrace{A_1\cos\alpha+A_2\cos 2\alpha+A_3\cos 3\alpha+\cdots+A_n\cos n\alpha}^{\substack{\text{cosine terms}\\ \downarrow}}$$
$$+\underbrace{B_1\sin\alpha+B_2\sin 2\alpha+B_3\sin 3\alpha+\cdots+B_n\sin n\alpha}_{\substack{\uparrow\\ \text{sine terms}}}$$

- Depending on the waveform few or large number of these terms may be required to estimate the waveform over one full cycle.

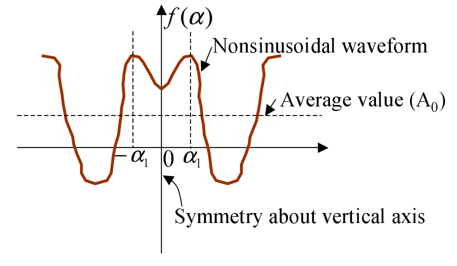
Symmetry about vertical axis:

$$\boxed{f(\alpha)=f(-\alpha)}$$

Even function: has axis symmetry

$$\Rightarrow \boxed{B_{1\to n}=0}$$

Function can be described by just a dc and cosine terms.



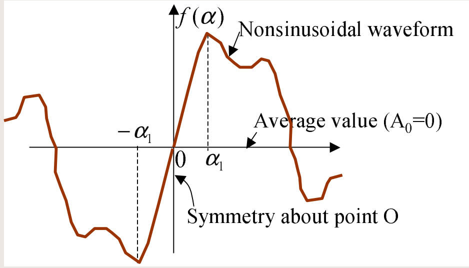
Point Symmetry:

$$\boxed{f(\alpha)=-f(-\alpha)}$$

Odd function: has point symmetry

$$\Rightarrow \boxed{A_{1\to n}=0}$$

Function can be described by just a dc and sine terms.



Fundamental component : first term of sine and cosine series: $\boxed{A_1 \cos\alpha_1 + B_1 \sin\alpha_1}$

Minimum frequency term required to represent the a particular waveform. It should be present in any Fourier Series representation.

Harmonic component : other terms of sine and cosine series:

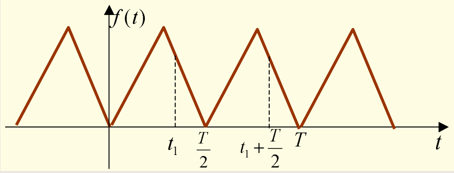
$\boxed{A_i \cos\alpha_i + B_i \sin\alpha_i \quad (1 < i \le n)}$

An element that has a frequency twice the fundamental is called the second harmonic; three times, the third harmonic; and so on.

If :

$$\boxed{f(t) = f\left(\frac{T}{2} + t\right)}$$

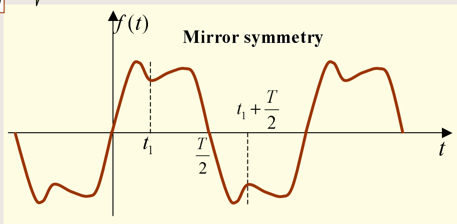
Odd harmonics of the series of cosine and sine terms are zero



If :

$$\boxed{f(t) = -f\left(\frac{T}{2} + t\right)}$$

Even harmonics of the series of cosine and sine terms are zero



Fourier constants are determined as follows:

$$\boxed{A_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2\pi}\int_0^{2\pi} f(\alpha)d\alpha}$$

$$\boxed{A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)dt = \frac{1}{\pi}\int_0^{2\pi} f(\alpha)\cos(n\alpha)d\alpha}$$

$$\boxed{B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)dt = \frac{1}{\pi}\int_0^{2\pi} f(\alpha)\sin(n\alpha)d\alpha}$$