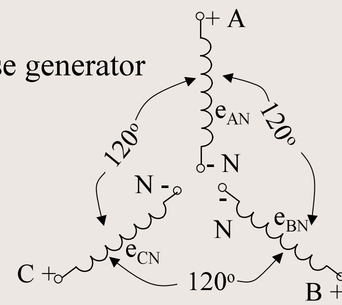
Balanced Three-Phase Systems

Three-Phase Generator

e_{AN}, e_{BN}, e_{CN}: Induced Voltages of 3-phase generator



$$e_{AN} = E_m \sin(\omega t)$$

$$e_{BN} = E_m \sin(\omega t - 120^o)$$

$$e_{CN} = E_m \sin(\omega t - 240^o)$$

The phasor diagram:

Phasor form of phase voltages

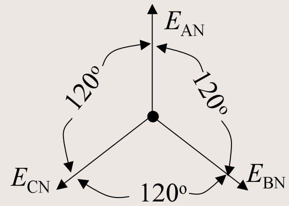
$$E_{AN} = E_{BN} = E_{CN} = E_{rms}$$

$$\overline{E}_{AN} = E_{rms} \langle 0^o$$

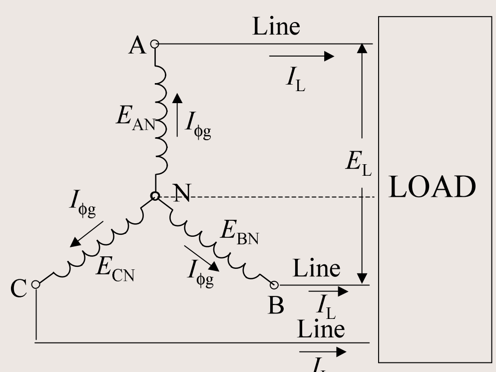
$$\overline{E}_{BN} = E_{rms} \langle -120^o$$

$$\overline{E}_{CN} = E_{rms} \langle +120^o$$

$$\overline{E}_{AN} + \overline{E}_{BN} + \overline{E}_{CN} = 0$$



The Y-Connected Generator



$$I_L = I_{\phi g}$$

$$E_L = \sqrt{3} E_\phi$$

$$E_{AB} = E_{AN} - E_{BN}$$

- The point at which all the points are connected is called the neutral point.
- Y-connected 3ϕ , 3-wire
- Y-connected 3ϕ , 4-wire (neutral connected)

Line voltage

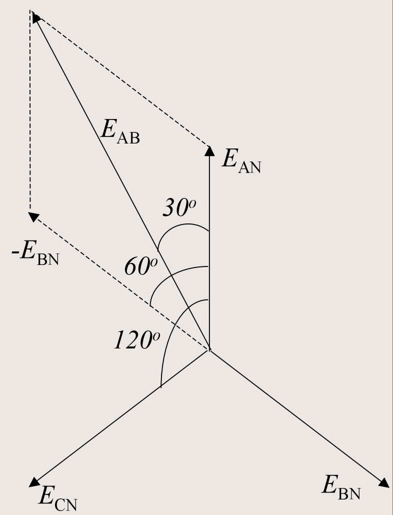
$$E_{AN} = E_\phi \langle 0^o \quad , \qquad E_{BN} = E_\phi \langle -120^o$$

$$E_{AB} = E_L = E_\phi \langle 0^o - E_\phi \langle -120^o$$

$$= E_\phi [1 + j0] - E_\phi \left[-0.5 - j\frac{\sqrt{3}}{2} \right]$$

$$= E_\phi \left[\frac{3}{2} + j\frac{\sqrt{3}}{2} \right]$$

$$E_L = \sqrt{3} E_\phi \left[\frac{\sqrt{3}}{2} + j\frac{1}{2} \right] = \sqrt{3} E_\phi \langle +30^o$$



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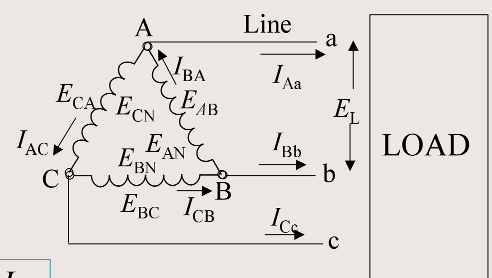
The Δ-Connected Generator

$$I_L = \sqrt{3} I_{\phi g}$$

$$E_L = E_\phi$$

$$I_{Aa} = I_{BA} - I_{AC}$$

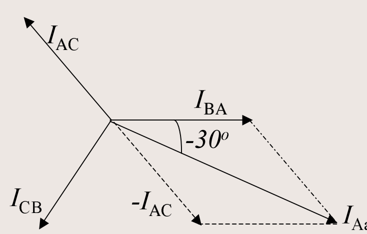


Proof: $I_{BA} = I_{AC} + I_{Aa} \Rightarrow I_{Aa} = I_{BA} - I_{AC}$

$$I_{BA} = I_{\phi g} \langle 0^o = I_{\phi g}(1 + j0)$$

$$I_{AC} = I_{\phi g} \langle 120^o = I_{\phi g}(-0.5 + j\frac{\sqrt{3}}{2})$$

Thus $I_{Aa} = \sqrt{3} I_{\phi g} \langle -30^o$



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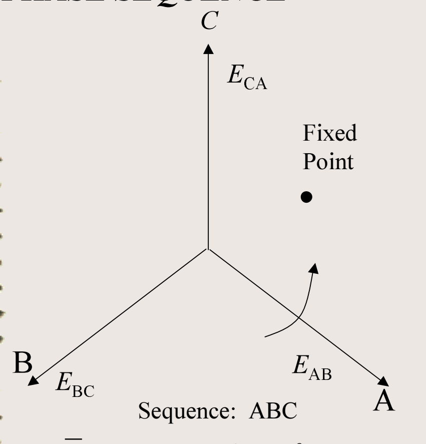
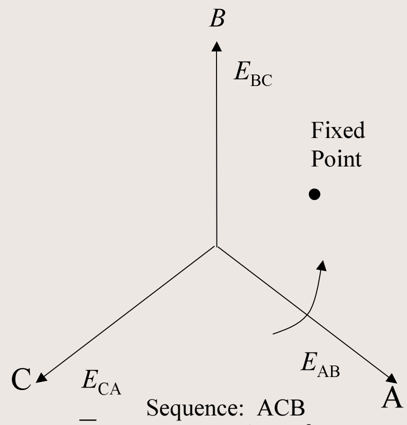
PHASE SEQUENCE





Sequence: ABC

$$\overline{E}_{AB} = E_{AB} \quad \angle 0 \quad reference$$
$$\overline{E}_{BC} = E_{BC} \quad \angle -120$$
$$\overline{E}_{CA} = E_{CA} \quad \angle +120$$

Sequence: ACB

$$\overline{E}_{AB} = E_{AB} \quad \angle\ 0 \quad referenc$$
$$\overline{E}_{CA} = E_{CA} \quad \angle -120$$
$$\overline{E}_{BC} = E_{BC} \quad \angle +120$$

Power In Three Phase Systems

Y-Connected balanced Load

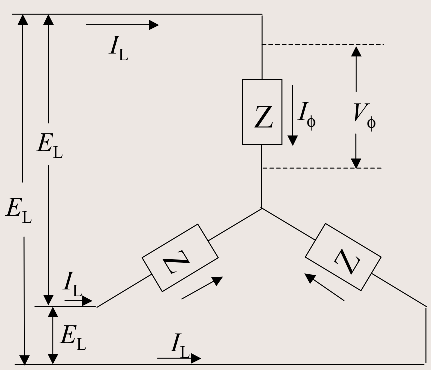
Average power delivered to each phase:

$$P_\phi = V_\phi I_\phi \cos\theta_{i\,I_\phi}^{V_\phi} = I_\phi^2 R_\phi$$

$$P_T = 3V_\phi I_\phi \cos\theta_{i\,I_\phi}^{V_\phi} = 3I_\phi^2 R_\phi$$

In this case $I_\phi = I_L$ and $V_\phi = \dfrac{V_L}{\sqrt{3}}$

$$\Rightarrow P_T = \sqrt{3} V_L I_L \cos\theta_{i\,I_\phi}^{V_\phi} = 3I_L^2 R_\phi$$



Δ-Connected balanced Load

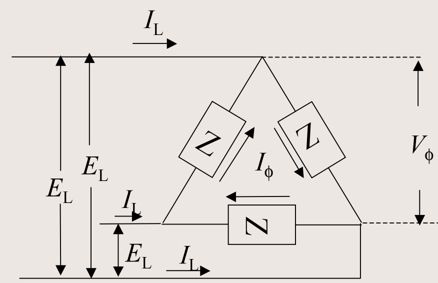
Average power delivered to each phase:

$$P_\phi = V_\phi I_\phi \cos\theta_{i\,I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_R^2}{R_\phi}$$

$$P_T = 3P_\phi$$

In this case $I_\phi = \frac{I_L}{\sqrt{3}}$ and $V_\phi = V_L$

$$\Rightarrow P_T = \sqrt{3} V_L I_L \cos\theta_{i\,I_\phi}^{V_\phi} = 3 I_L^2 R = \frac{3V_R^2}{R_\phi}$$



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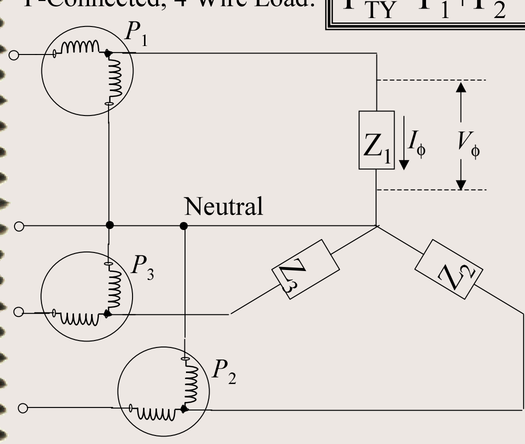
Power Measurement In 3 Phase Systems

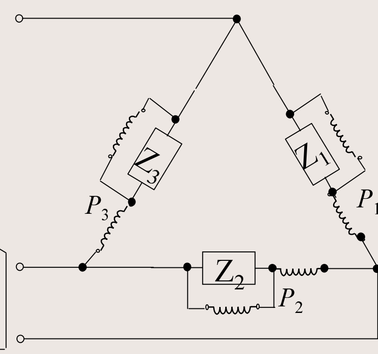
The 3 Wattmeter Method

Y-Connected, 4 Wire Load:

$$P_{TY} = P_1 + P_2 + P_3$$

$$P_{T\Delta} = P_1 + P_2 + P_3$$





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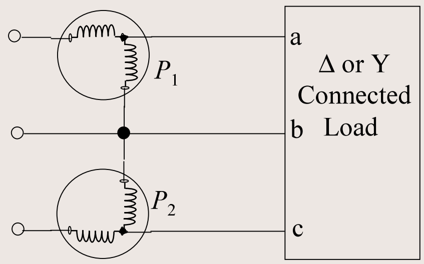
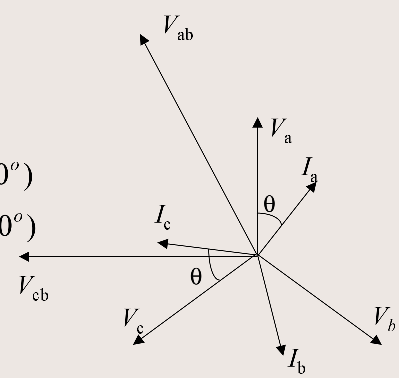
The 2 Wattmeter Method



$P_T = P_1 + P_2$



$$P_1 = I_a V_{ab} \cos\theta_1 = V_L I_L \cos(\theta + 30^o)$$

$$P_2 = I_c V_{cb} \cos\theta_2 = V_L I_L \cos(\theta - 30^o)$$

$$P_T = P_1 + P_2 = \sqrt{3} V_L I_L \cos(\theta)$$

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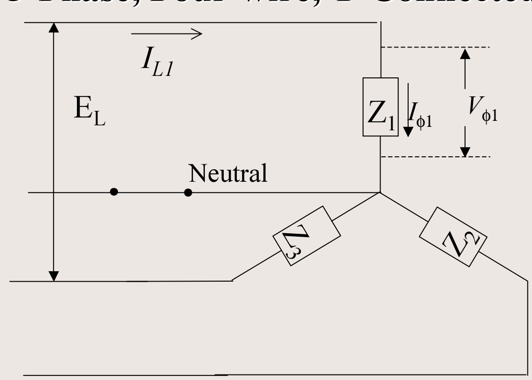
Unbalance 3-Phase, Four-wire, Y-Connected Load



$$V_\phi = E_\phi, \quad I_{\phi 1} = I_{L1} = \frac{V_{\phi 1}}{Z_1}, \quad I_N = I_{\phi 1} + I_{\phi 2} + I_{\phi 3}$$

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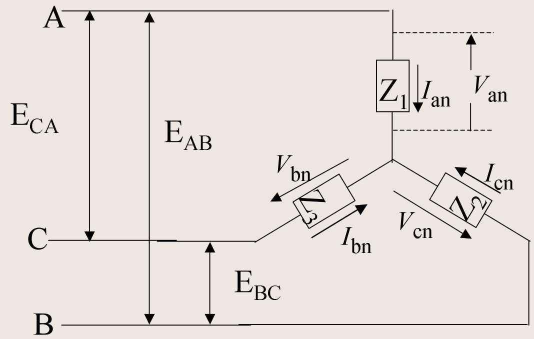
Unbalance 3-Phase, 3-wire, Y-Connected Load



$$I_{an} = \frac{E_{AB}Z_3 - E_{CA}Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3I_{an}} \qquad I_{bn} = \frac{E_{BC}Z_1 - E_{AB}Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$I_{cn} = \frac{E_{CA}Z_2 - E_{BC}Z_1}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

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