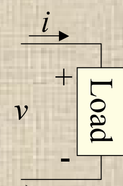
AVERAGE POWER

The instantaneous power to the load is $P = v\,i$



In general: $v = V_m \sin(\omega t + \theta)$ and $i = I_m \sin(\omega t)$

Therefore,

$$p = [\frac{V_m I_m}{2}\cos(\theta)] - \frac{V_m I_m}{2}\cos(\theta)\cos(2\omega t) + \frac{V_m I_m}{2}\sin(\theta)\sin(2\omega t)$$

The first term in the preceding equation has a constant magnitude (no time dependence) and therefore provides some net transfer of energy.

This term is referred to as the *average Power or real Power*.

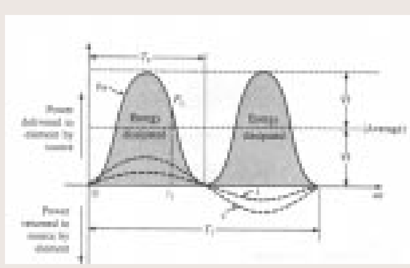
since $V_{eff} = \frac{V_m}{\sqrt{2}}$ *and* $I_{eff} = \frac{I_m}{\sqrt{2}}$ ⟹ $P = V_{eff} I_{eff} \cos\theta$

Resistor

V and I are in phase ⟹ $\theta = 0$

$$\therefore\ P = V_{eff} I_{eff} = \frac{V_{eff}^2}{R} = I_{eff}^2 R = I^2 R$$

The energy dissipated by the resistor over one full cycle of the power curve: $P = V_{eff} I_{eff} T_2 = V\,I\,T_2$



Inductor

I lags V by 90° → $\theta = 90$

$$\therefore \; \boldsymbol{P} = \boldsymbol{V}_{eff} \boldsymbol{I}_{eff} \cos(90) = 0$$

Capacitor

I leads V by 90° → $\theta = 90$

$$\therefore \; \boldsymbol{P} = \boldsymbol{V}_{eff} \boldsymbol{I}_{eff} \cos(90) = 0$$

POWER FACTOR

$$Power\ factor = F_p = cos\,\theta = \frac{P}{VI}$$

The power factor can be leading or lagging.

- The more reactive a load, the lower the power factor, and the smaller the average power delivered.
- Low power factor are usually avoided, since a high current would be required to deliver any appreciable power. This higher current demand produces higher heating losses and, consequently, the system operates at a lower efficiency.
- Capacitive networks have leading power factors, and inductive networks have lagging power factors.

APPARENT POWER

It is the product of voltage and current, it units are *volts-ampers*. Its magnitude is determined by:

$$S = VI$$

The average power is defined by: $P = VI\cos\theta$

Therefore, $P = S\cos\theta$

Or the power factor of a system is $F_p = \cos\theta = \frac{P}{S}$

REACTIVE POWER

The reactive power is defined as:

$$Q = VI\sin(\theta)$$

For Inductor, $\theta = 90$

$$Q_L = VI = I^2 X_L = \frac{V^2}{X_L}$$

$$W_L = LI^2$$

For Capacitor, $\theta = 90$

$$Q_c = VI = I^2 X_c = \frac{V^2}{X_c}$$

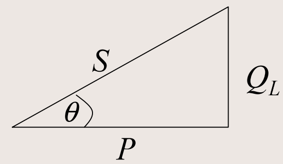
$$W_L = CV^2$$

THE POWER TRIANGLE

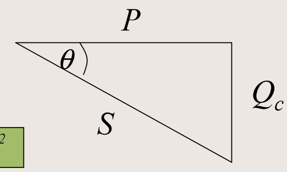
The average power, apparent power and reactive power can be related by

$$S = P + JQ$$

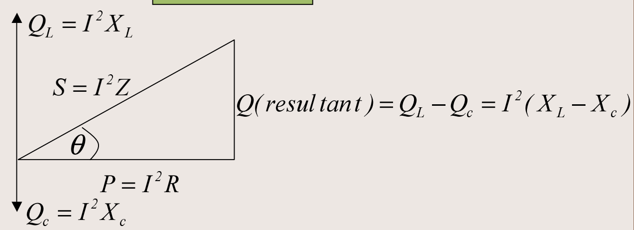
For an inductive load $S = P + JQ_L$



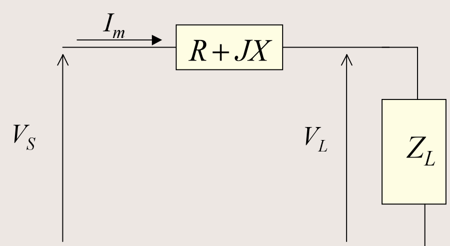
For a capacitive load $S = P - JQ_c$



The three powers are related by the Pythagorean theorem; $S^2 = P^2 + Q^2$



POWER FACTOR CORRECTION



V_L is constant

Power absorbed by the line (P losses) = $I_m^2 R$

Power delivered to the load $= P_L = V_L I_m \cos(\theta)$

$$I_m = \frac{P_L}{V_L \cos(\theta)} \Longrightarrow \cos(\theta)\uparrow \Rightarrow I_m \downarrow \Rightarrow P\ Losses \downarrow$$

How could we improve the power factor? Usually by connecting a capacitor in parallel with the loads.

The process of introducing reactive elements to bring the power factor closer to unity is called power-factor correction.