PS160 Review Problems

1. A flywheel with moment of inertia 20 $kg - m^2$ and radius $R = 0.5$ *meters* is initially turning at 30 rad/s. It is slowed by a braking torque to 10 rad/s in four seconds. What is the angular acceleration?

Solution: Use the basic rotational equations.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{10 - 30}{4 - 0} = -5\ rad/s^2$$

2. Same problem as in number 1. What is the angular velocity after three seconds?

Solution: Use the basic rotational equations again.

$$\omega = \alpha t + \omega_0 = -5t + 30 \mid_{t=3} = 15\ rad/s$$

3. Same problem as in number 1. What is the magnitude of the braking torque, if it is applied tangent to the rim? Assume the braking torque is constant with time.

Solution: Now we use the torque equivalent of Newton's second law.

$$\tau = FR = I\alpha \Rightarrow F = \frac{I}{R}\alpha = \frac{20}{0.5} \cdot (-5) = -200\ Newtons$$

4. A block is attached to a pulley at the top of a ramp, and is allowed to slide down the ramp as the cable unravels. What is the speed of the block at the bottom? Use conservation of energy.

Solution:

$$\Delta K + \Delta K_{rot} + \Delta U = 0$$

$$K_f - K_i + K_{rot_f} - K_{rot_i} + U_f - U_i = 0$$

$$\frac{1}{2}mv^2 - 0 + \frac{1}{2}I\omega^2 - 0 + 0 - mgh = 0$$

Now use the fact that $v = R\omega$.

$$\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 - mgh = 0$$

Rearrange and combine the two velocity terms.

$$\frac{1}{2}v^2\left(m + \frac{I}{R^2}\right) = mgh$$

If the pulley is considered a disk, its moment of inertia is given by

$$I_{disk} = \frac{1}{2}mR^2$$

Plugging this in, get

$$\frac{1}{2}v^2\left(m+\frac{1}{2}m\right)=\frac{3}{4}mv^2=mgh$$

Solve for v:

$$v=\sqrt{\frac{4gh}{3}}$$

5. See problem four. Find the tension in the rope.

Solution: This is fairly challenging, and requires Newton's laws together with the torque equation. In the process, problem 6 will also be solved. Use the following equations. Assume the positive x-axis slopes down to the right, and the block goes that direction, and that the y-axis is perpendicular to the surface of the slope. Then the following equations hold:

$$R\alpha=-a$$

This is because, with this setup, the acceleration is in the positive x-direction but the pulley spins clockwise,which is the negative angular direction.

$$x:\quad ma_x=ma=mg\sin\theta-T$$

$$y:\quad ma_y=0=N-mg\cos\theta$$

$$\tau:\quad I\alpha=-TR$$

The τ equation becomes

$$Ia=TR^2$$

after the substitution. Solve this last equation for T, plug it into the x-equation, and solve for the acceleration. Then back substitute one more time to get the tension in the string, and you've solved five and six.

6. See problem four. Find the acceleration of the block.

7. A plank lies across two supports as shown. The plank masses 20 kilograms and is uniform. A boy massing 40 kg walks towards the end of the plank. At what point will the plank just begin to tip up?

8. A merry-go-round massing 400 kilograms and radius 3 meters is turning at 10 radians per second. Two kids, each massing 50 kilograms, step on to the rim opposite each other. What is the new angular velocity of the merry-go-round?

9. Refer to the previous problem. The kids start walking towards each other and stand one-half of a meter from the center. What is the angular velocity?

10. A rocket is in circular orbit around the earth, 250 kilometers up. What Delta V is needed to put the rocket on a new orbit, with apogee at the moon? The moon is about 380,000 kilometers away.

Solution: This requires solving two equations and two unknowns. The unknowns are the velocities at perigee (closest to Earth) and apogee (farthest from Earth). The two equations are the conservation of energy equation and the conservation of momentum equation.

$$K_i + U_i = K_f + K_i$$

$$\frac{1}{2}mv_p^2 - \frac{mMG}{r_p^2} = \frac{1}{2}mv_a^2 - \frac{mMG}{r_a^2}$$

$$L_i = L_f$$

$$mr_pv_p = mr_av_a$$

Notice that here we're taking advantage of the fact that at perigee and apogee the velocity vector is perpendicular to the position vector, so the angle between them is ninety degrees and $\sin 90^o = 1$. After some simple algebra, find the answer.

11. A rocket blasts straight up from the moon at 1000 m/s. How far up does it go before falling back? Neglect the boost phase.

Solution: This is easier than the previous problem. Use conservation of energy only.

$$K_i + U_i = K_f + K_i$$

$$\frac{1}{2}mv_i^2 - \frac{mMG}{r_i^2} = \frac{1}{2}mv_f^2 - \frac{mMG}{r_f^2}$$

12. In a hydrogen atom, light of so many nanometers is observed. Assuming this is due to a transition to the ground state, what was the value of n for the electrons making this transition?

Solution: For these kinds of problems (this one is not stated too accurately), you can often get by with:

$$E = hf = h\frac{c}{\lambda}$$

and with

$$E_n == \frac{-13.6\ eV}{n^2}$$

13. A 5 kg mass on a horizontal spring is shoved to the left at 2 m/s. What is the amplitude, assuming the spring constant is 400 N/m?

Solution: For this, use conservation of energy. The initial speed corresponds to the kinetic energy at the origin, which goes to zero at maximum displacement, i.e.

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}kA^2$$

14. Refer to number 13. At what point is the mass going 1 m/s?

Solution: Use conservation of energy again. Compare the point where the block is going 1 meter per second either to the point of maximum displacement (where the kinetic energy is zero), or minimum displacement (the origin, where the potential energy is zero). Solve for the displacement.

15. Refer to number 13. Which of the following best describes the motion of the mass mathematically?

Solution: Start with

$$x = A\cos(\omega t + \delta)$$

and

$$v = -A\omega\sin(\omega t + \delta)$$

Get ω from

$$\omega = \sqrt{\frac{k}{m}}$$

The amplitude was found in a previous problem. Get the phase shift from the initial conditions at t=0:

$$x = 0 = A\cos\delta$$

$$v = -2 = -A\omega\sin\delta$$

The first equation gives $\delta = \pi/2 \quad ; 3\pi/2$ From the second, it is evident that the choice $\pi/2$ is the correct one.