Chapter 15: Temperature and Heat

Objectives: Here, we'll learn about the difference between temperature and heat, and how matter expands and contracts when the temperature changes. In addition, we'll find out about heat capacity, and how to solve problems involving heat transfer from one object to another. We'll learn about phase changes, and what kind of heat transfers have to take place for such phase changes to happen. Finally, we'll learn about the three ways heat goes from one place to another: conduction, convection, and radiation.

15.1 Temperature

Temperature is a measure of the activity of the atoms in a given body. It isn't the same as heat. A plasma, for example, can be at a very high temperature, but contain very little heat energy, just because the plasma has a very low density. Water can absorb and carry a lot of heat energy, even when at a relatively low temperature, making the climates of islands and peninsulas milder by absorbing heat energy on hot days, and letting it out slowly on cool days.

There are three temperature scales in common use. These are: the ***Fahrenheit Scale***, the ***Celsius Scale***, and the ***Kelvin*** or ***Absolute Scale.***

The ***Fahrenheit scale*** chooses the freezing point of water to be at ***32 degrees***, and the boiling point at ***212 degrees***, under a pressure of one atmosphere. Since the freezing point and boiling point are affected by the pressure, we have to specify this standard pressure. The temperature scale points are divided up evenly between these two points, and extended above and below.

The ***Celsius Scale***, formerly called ***Centigrade Scale,*** puts the freezing point of water at ***0^o C*** and the boiling point at ***100^o C.*** Again, the scale markings, called ***hache marks***, are evenly divided, and extend above the boiling point and below the freezing point.

The ***Kelvin Scale*** has hache marks the same distance apart as the Celsius scale. However, it starts measuring temperatures at ***absolute zero***, which corresponds to ***-273.15^o Celsius***. This temperature was discovered by making pressure versus temperature graphs for several different gases. As the gases were cooled, they created straight lines of data in the P-T plane. These straight lines, with difference slopes, all converged together at one unique point, ***-273.15^o Celsius***. At this point, the gas pressure goes effectively to zero, and all motion ceases.

To convert from Celsius to Kelvin, use the following simple formula:

$$^oK = \ ^oC + 273.15^o$$

The inverse of this equation is pretty obvious–just subtract 273.15 from both sides. To convert from Celsius to Farenheit, use

$$C = \frac{5}{9}(F - 32)$$

while the inverse relationship is

$$F = \frac{9}{5}C + 32$$

These equations can be figured out just by graphing the freezing and boiling points on a graph of F versus C, then finding the equation of the line connecting them.

Example 1. Convert 58^o C. to (A) Kelvin and (B) Fahrenheit.

Solution: Plug into the equations.

$$^o K = {}^o C + 273.15^o = 58 + 273.15 = 331.15^o K$$

$$F = \frac{9}{5}C + 32 = \frac{9}{5} \cdot 58 + 32 = 136.4^o F$$

15.2 Thermal Expansion

As objects warm, they usually expand. This is because the atoms absorb energy, and either stretch the bonds just as a spring might, or bounce against each other with greater force, resulting in a greater intermolecular spacing. Macroscopically, for engineering applications, this is very easy to model with the following equation:

$$\Delta L = \alpha L_o \Delta T$$

ΔL is the length, L_o the original length before heating, ΔT is the temperature difference, which can be positive or negative, and α is called the coefficient of thermal expansion. A similar equation holds for volume expansion:

$$\Delta V = \beta V \Delta T$$

Naturally, for areas there is yet another equation, with the obvious replacements, though traditionally it does not appear in texts. It can be easily shown via either algebra or calculus that $\beta = 3\alpha$. A similar equation would hold for areas, with a constant $\gamma = 2\alpha$. These linear relationships hold only for a certain temperature range, since a substance could change phase! In addition, some substances can expand with lower temperature, such as water just before freezing. If this were not the case, ice wouldn't be less dense than water, wouldn't float, and would tend to accumulate on the bottom of the ocean, eventually resulting in an ice-locked planet like Hoth in the Star Wars movie "The Empire Strikes Back".

Example 2. A rod of length 2 meters expands by one millimeter when the temperature is raised by 400 degrees Celsius. What is the coefficient of linear expansion?

Solution: This is a matter of plugging in and solving for the coefficient.

$$\Delta L = \alpha L \Delta T \quad \Rightarrow \quad \alpha = \frac{\Delta L}{L \Delta T} = \frac{1 \times 10^{-3}}{2 \cdot 400} = 1.25 \times 10^{-6} \ {}^{\circ}C^{-1}$$

Example 3. A steel can with volume 0.1 cubic meter is filled to the brim with ethyl alcohol, all initially at 20 °C. The can and alcohol are then warmed to 60 °C. How much alcohol spills out of the can?

$\beta_{steel} = 36 \times 10^{-6} \ \ {}^{\circ}C^{-1}$; $\beta_{alco} = 1120 \times 10^{-6} \ \ {}^{\circ}C^{-1}$

Solution: This is a trick question. The trick is, the can expands as well as the alcohol, so less will be spilled than thought. Of course, there are details to consider, such as which substance expands faster, but those details will be ignored. Also, the volume equation still works for the can, even though the can is not filled in. Again, some details must be ignored.

$$\Delta V_{alc} = \beta_{alc} V \Delta T = 1120 \times 10^{-6} \cdot 0.1 \cdot (60-20) = 4.48 \times 10^{-3} \ m^3$$

$$\Delta V_{can} = \beta_{st} V \Delta T = 36 \times 10^{-6} \cdot 0.1 \cdot (60-20) = 1.44 \times 10^{-4} \ m^3$$

$$volume \ \ spilled = 4.48 \times 10^{-3} - 1.44 \times 10^{-4} = 4.34 \times 10^{-3} \ m^3$$

So the volume spilled is a little bit less than expected, because of the expansion of the can.

15.3 Heat Capacity

The heat energy absorbed by a substance for a given change in temperature depends on the fine details of its atomic and molecular structure. For most purposes, these details can be all accounted for in an empirical quantity called the specific heat capacity, ***c***. We therefore write:

$$Q = m\,c\,\Delta T$$

where ***Q*** is the heat energy in joules, ***m*** is the mass, ***c*** is the heat capacity, and ***T*** is the temperature. A similar idea is that of the molar heat capacity, usually denoted by ***C***. Heat is also measured in ***calories***, or ***kilocalories***. One kilocalorie equals 4,186 Joules. Another common unit, on your monthly electric bill, is the ***kilowatt-hour*** , which equals 3.6 x10^6 J.

Example 4. Find the heat necessary to warm 5 kilograms of water from a temperature of 25 degrees Celsius to the boiling point of 100 degrees Celsius. $c_{H_2O} = 4{,}186 \quad J/kg{-}°C$

Solution: Plug and chug. Since it’s a temperature difference, we don’t need to convert to Kelvin. Temperature differences are the same in both units.

$$Q = m\,c\,\Delta T = 5 \cdot 4{,}186 \cdot (100 - 25) = 1.57 \times 10^6 \quad Joules$$

Example 5. Into a 0.2 kg copper cup at 20 C put 0.100 kg of aluminum at 50.0 C and 0.250 kg of water at 85.0 C . Assuming there is no heat exchange with the outside world, find the final equilibrium temperature.

Solution: To avoid arithmetic errors in more elaborate problems (more than two substances), it’s best to make a table, then use the fact that the sum of the heat transfers must equal 0.

	m	c	T_f	T_i	Q
Copper	0.2	387	T	20	$m\,c\,\Delta T = 77.4\,(T - 20)$
Aluminum	0.2	900	T	50	$m\,c\,\Delta T = 180\,(T - 50)$
Water	0.25	4,186	T	85	$m\,c\,\Delta T = 1{,}046.5\,(T - 85)$

Finally, sum up the heat transfers:. Note how water dominates, due its large heat capacity.

$$Q_{Cu} + Q_{Al} + Q_{H_2O} = 0$$

$$77.4\,(T - 20) + 180\,(T - 50) + 1046.5\,(T - 85) = 0$$

$$\left(77.4 + 180 + 1046.5\right)T - \left(77.4 \cdot 20 + 180 \cdot 50 + 1046.4 \cdot 85\right) = 0$$

$$1.304 \times 10^3\,T = 9.95 \times 10^4 \quad \Rightarrow \quad T = 76.3 \quad °C$$

15.4. Phase Transitions

During phase transitions, such as ice to water or water to steam, the temperature remains constant, but a change of state occurs. This change of state requires a change in energy, hence an influx or efflux of heat. Ice at zero degrees Celsius, for example, requires heat to become water at zero degrees Celsius.

The amount of heat exchange needed is modeled by two constants. The first of these is the heat of fusion, L_f, which helps us calculate how much heat is required to make a solid turn into a liquid, when it's at the melting point. This constant is found by experiment. This heat is given by

$$Q = m L_f$$

The second is the heat of vaporization , L_v, which tells us how much heat is necessary to take a substance from a liquid at the boiling point to a gas at the same temperature. Again, experiments tell us what these constants are for difference substances. The formula for the heat is the same as before:

$$Q = m L_v$$

You have to be careful about assigning either a plus or minus sign to either of these terms–minus when the substance is losing heat, and plus when it's gaining heat.

Example 6. How much heat is required to turn five kilograms of water, boiling at 100 C., to steam at 100 C.? Note: the heat of vaporization for water is $2.26 \times 10^6\ J/kg$.

Solution: This is easy. Just plug in.

$$Q = m L_v = 5 \cdot 2.26 \times 10^6 = 1.13 \times 10^7 \quad J$$

Example 7. A 10.0 kg block of ice has a temperature of -20.0 C. The block absorbs 1500 kcal of heat energy. What is the final temperature of the water? Note: ice has a specific heat of 0.478 kilocalories per kg-C. Water has a specific heat of 1 kilocalorie per kg-C.

Solution: $\boldsymbol{Q_1}$ is the heat required to warm the block to 0 degrees C., $\mathbf{Q_2}$ is the heat required to melt the ice at 0 degrees, and $\mathbf{Q_3}$ is the heat that warms the melted ice to its final temperature. Naturally, you have to be sure that the ice actually melts, or else $\mathbf{Q_3}$ doesn't enter the equation!

$$Q_1 + Q_2 + Q_3 = 1500 \quad kcal$$

$$Q_1 = m c \Delta T = 10 \cdot 0.478 \cdot (0 - (-20)) = 95.6 \ kcal$$

$$Q_2 = m L_f = 10 \left(33.5x10^4 \ \frac{J}{kg} \right) \left(\frac{1 \ kcal}{4{,}186 \ J} \right) = \frac{3.35 \times 10^6}{4{,}186} = 800.3 \quad kcal.$$

So it takes 95.6 kcal to warm the ice to 0 degrees C, and 800.3 kcal to completely melt the ice. This plus $\mathbf{Q_3}$ adds up to 1500 kcal:

$$895.9 + Q_3 = 1500 \quad kcal$$

$$\Rightarrow \quad Q_3 = 1500 - 895.9 = 604.1 \quad kilocalories = m c \Delta T = (10) \cdot 1 \cdot (T - 0)$$

$$\Rightarrow \quad \Rightarrow \quad T = 60.4 \ °C$$

15.5 Heat Transfer

There are three ways of transferring heat: ***conduction, convection,*** and ***radiation***.

In ***conduction***, heat is transferred by molecules in a hot spot jostling the molecules next to them. A typical example is found in the kitchen. Metal pots usually have wood or other material as a handle, because a metal handle will soon become hot even though it's not in contact with the stove top. Problems involving conduction are solved with the heat equation. The rate, ***H***, at which heat is transferred is given by

$$H = \frac{k A \Delta T}{L}$$

k = thermal conductivity, an experimentally obtained quantity
A = the area through which the heat is passing
ΔT = the difference in temperature between inside the region and outside the region
L = the length, or thickness, through which the heat must be transferred.

Example 8. Body fat has a thermal conductivity of k=0.2 J/s-m-C. The area of a human body might be approximately 2 m^2. The thickness of fat beneath the skin varies quite a bit, but let's say it's about a centimeter, on the average. At what rate is power radiated from the body, if the ambient temperature is 25 C.?

Solution: Humans have temperatures of 98.6 ^{o}F. Convert this to Celsius.

$$C = \frac{5}{9}(F - 32) = \frac{5}{9}(98.6 - 32) = 37\ °C$$

Now put all the numbers into the equation for heat conduction.

$$H = \frac{k A \Delta T}{L} = \frac{0.2 \cdot 2 \cdot (37 - 25)}{0.01} = 480\ watts$$

That's a lot of energy to be losing every second, which is why it's advisable to wear clothing. Also, it's possible to die of exposure even in warm equatorial waters—due to heat loss---cooling to death.

In ***convection***, heat is transferred by moving a heated mass, of water, from a hot spot to a cooler spot. Convection cells set up in pots of hot water is a good example, and these can be seen. Currents of hot water go from the bottom of the pot to the top and circulate back down again, in a loop. This is in contrast to conduction, where atoms stay mainly in the same area, and transfer heat by disturbing their neighbors.

Heat can also be transferred by ***radiation***. This radiation is in the form of ***photons***, which are particles of light. This is described by the ***Stefan-Boltzman equation***.

$$P = \sigma A e T^4$$

P = Power radiated
$\sigma = 5.67 \times 10^{-8}\ watts/m^2\ °K^4$—this is the Stefan-Boltzman constant
A = the area of the radiating surface
e = emissivity of the surface, and takes values between 0 and 1.
T = temperature ***in Kelvin only.***

Emissivity is just a measure of the tendancy for a body to emit light. The sun is an almost perfect emitter, so its emissivity is nearly 1.

Example 9. A ball of radius 2 meters and emissivity 0.8 is heated to a temperature of 400 degrees Celsius and placed in a vacuum. The walls of the chamber are perfectly absorbing and maintained at absolute zero. At what rate does the ball emit energy?

Solution: The walls of the chamber have to be at absolute zero and perfectly absorbing or else radiation from them–either emitted or reflected—will encounter the sphere, which will absorb some of it. This isn't a show-stopper, it would just make the problem a lot more complicated.

$$P = \sigma A e T^4 = 5.67 \times 10^{-8} \cdot \left(4\pi \cdot 2^2\right) \cdot 0.8 \cdot (273 + 400)^4 = 4.68 \times 10^5\ watts$$

Notice that we had to convert to Kelvin before proceeding.

15.6 More Examples

Example 10. If the price of electrical energy is $0.25 per kilowatt-hour, what is the cost of using electrical energy to heat the water in a swimming pool (20.0 m x 10.0 m x 3 m) from 15 to 30 degrees Celsius?

Solution: We know **c** and ΔT : we must get $\boldsymbol{m}$, the mass of water in the pool. The volume is found by just multiplying the length times height times width, 20 x 10 x 3 = 600 m^3.

$$m = \rho V \quad \Rightarrow \quad m = 1000\ \frac{kg}{m^3} \cdot 600\ m^3 = 600{,}000\ \ kg$$

Next we find the amount of heat, $\boldsymbol{Q}$, needed to raise the temperature from 15 C to 30 C.

$$Q = m\,c\,\Delta T = 600{,}000 \cdot 4{,}186 \cdot (30 - 15) = 3.77 \times 10^{10} \quad Joules$$

That's a lot of joules. Next we have to convert this to an equivalent number of kilowatt-hours. A kilowatt-hour is an amount of energy, not a power. It is equal to one thousand watts (Joules/second) times 3600 seconds (the number of seconds in an hour).

$$1 \ kw-hr = 1000 \ watts \cdot 3600 \ \sec = 3.6 \times 10^6 \quad Joules$$

Dividing $\boldsymbol{Q}$ by this number gives the number of kilowatt hours needed. Multiplying by 0.25/ kilowatt-hrs gives the total cost.

$$\frac{3.77 \times 10^{10} \ J}{3.6 \times 10^6 \quad J/kw-hr} = 1.05 \times 10^4 \ kw-hr$$

$$\Rightarrow \quad Cost = 1.05 \times 10^4 \ kw-hr \times \frac{0.25}{kw-hr} = \$\,2{,}618.06$$

The last digits are not significant, since a lot of rounding went into the problem.

Example 11. During exercise, sweat forms on the skin and evaporates, helping the body get rid of heat. The latent heat of vaporization of H_2O at body temperature, 37 C. , is $2.42x10^6$ J/kg. To cool the body of a 80 kg jogger by 1.0 C, how many kilograms of water in the form of sweat have to be evaporated? The specific heat of the human body can be taken to be $c_{bod} = 3{,}500 \quad J/kg-C$

Solution: The heat lost by the jogger plus the heat gained by the sweat must equal zero. Plug and chug.

$$Q_{bod} + Q_{sweat} = 0$$

$$M_{bod}\,c_{bod}\,\Delta T + M_{sweat}\,L_v = 0$$

$$80 \cdot 3{,}500 \cdot (-1) + M_{sweat} \cdot 2.42 \times 10^6 = 0$$

$$M_{sweat} = 0.116 \ kg$$

Example 12. It is claimed that if a lead bullet goes fast enough, it can melt completely when it comes to a halt suddenly and all of its kinetic energy is converted into heat via friction. Find the minimum speed of a lead bullet, initially at 30.0 C, for such an event to happen. Note: for lead, $c = 128\ J/kg{-}°C$ and the latent heat of fusion is $L_f = 2.32 \times 10^4\ J/kg$. The melting point of lead is 327.3 degrees C.

Solution: First, compute the heat energy required to melt a lead bullet at 30 degrees C. There will be two Q's: one to warm it to the melting point and the other to actually melt it. Add these, then set this equal to the kinetic energy, and solve for v.

$$\frac{1}{2}mv^2 = Q = m\,c\,\Delta T + m\,L_f$$

$$v^2 = 2\,c\,\Delta T + 2\,L_f$$

$$v = \sqrt{2\,c\,\Delta T + 2\,L_f} = \sqrt{2 \cdot 128 \cdot (327.3 - 30) + 2 \cdot 2.32 \times 10^4} =$$

$$= \sqrt{1.225 \times 10^5} = 350\ \frac{m}{s}$$

Example 13. Suppose 10,000 kilograms of ice at -20 C. are thrown in a beluga whale tank which is five meters deep and and has surface area of 100 m^2. If the temperature is initially 15 C., what common temperature do the ice and water finally reach, assuming no significant gain or loss of heat from the environment?
Specific heats: $c_{ice} = 2000\ J/kg{-}C \qquad c_{H_2O} = 4186\ J/kg{-}C$
heat of fusion, water: $L_f = 3.35 \times 10^5\ \ J/kg$

Solution: (A) Part A is straight forward. There are four terms: the heat necessary to warm the ice from -20 C to 0 C, the heat necessary to melt the ice, the heat necessary to warm the melted ice up to the final unknown temperature, T, and the change in temperature of the liquid water already in the tank.

$$m_{ice}\,c_{ice}\big(0-(-20)\big) + m_{ice}\,L_f + m_{ice}\,c_{H_2O}\big(T - 0\big) + m_{H_2O}\,c_{H_2O}\big(T-15\big) = 0$$

Next, we have to find out what mass of water there was, originally. Use the density, which is 1000 kg/m^3.

$$\rho = \frac{m}{V} \quad \Rightarrow \quad m = \rho\,V = 1000 \cdot \big(100 \cdot 5\big) = 500{,}000\ kg$$

The rest follows easily by plugging in the numbers and solving for ***T***.

$$m_{ice}\, c_{ice}\left(0-(-20)\right) \;+\; m_{ice} L_f \;+\; m_{ice}\, c_{H_2O}\left(T \;-\; 0\right) \;+\; m_{H_2O}\, c_{H_2O}\left(T-15\right) \;=\; 0$$

$$10{,}000 \cdot 2000 \cdot 20 \;+\; 10{,}000 \cdot 3.35 \times 10^5 \;+\; 10{,}000 \cdot 4186 \cdot T \;+\; 500{,}000 \cdot 4186 \cdot \left(T \;-\; 15\right) \;=\; 0$$

$$4 \times 10^8 \;+\; 3.35 \times 10^9 \;+\; 4.186 \times 10^7\, T \;+\; 2.093 \times 10^9\, T - 3.1395 \times 10^{10} \;=\; 0$$

$$2.135 \times 10^9\, T \;=\; 2.7645 \times 10^{10}$$

$$T \;=\; 12.95 \;\; °C$$

Not hard, except for stubbing our fingers on the buttons of our calculators. So ten metric tons of ice only give a couple degrees of relief to the overheated mammals.