Chapter 5: FORCES

Objectives: This chapter is basically a continuation of the last chapter, except more forces are introduced (or reintroduced) and some of the problems are harder. There are several different kinds of important forces. The most common forces are contact forces, like friction, the normal force, and air drag. Then there are the four fundamental forces of nature: the strong force, the electromagnetic forces, the weak nuclear force, and gravitation. These will be briefly introduced here, and studied at length in later chapters. Also introduced here are different types of common problems and techniques for their solution. At the end of this chapter, you will have a good mastery of all these ideas and techniques.

5.1 Weight and the Normal Force

Mass is not the same as weight. Mass is an intuitive quantity, something everyone knows, or has a feel for, and is a measure of the amount of stuff something is made of. It's the result of numerous fields and forces, and is not well understood.

Weight is given by mass times the local acceleration of gravity, . It's a force, and has a direction. You weigh less on the moon than on the Earth, but your mass is the same everywhere. The acceleration of gravity near Earth's surface is called g, and is equal to 9.8 m/s^2 , or in British units, 32 ft/s^2 .

The ***normal force*** is what keeps things from falling through other things. For example, it prevents us from falling through the surface of the Earth. It represents the effect of primarily electromagnetic forces that act between atoms, giving what is actually mostly empty space a feeling of solidity. The normal force is called such because it acts perpendicular to the given surface. In math, perpendicular and normal have much the same meaning.

Example 1. A block of mass ***m*** sits on a table. Find the normal force on the block.

Solution: Gravity and the normal force are acting on the block. The block isn t moving, so the acceleration is zero, and Newton s second law gives

So the normal force is often (but not always) a reaction force to gravity.

5.2 Friction Forces

Friction is a contact force that arises as a result of roughness of the surfaces in contact. Microscopic imperfections and transient bonds will create a resistance force that always acts opposite the direction of motion. There are two types of friction: ***static friction*** and ***kinetic friction.***

5.2.1 Static Friction

Static friction is a friction force on an unmoving object., and is modeled by a number called the ***coefficient of static friction***, that is obtained experimentally, times the normal force. The static friction force is given by

where is called the coefficient of static friction, and ***N*** is the normal force. The static friction force always opposes the force trying to create the motion. The normal force is the force that prevents the object from falling through the table or floor, and is provided by the particles and fields making up whatever the object is resting on.

Example 2. A force of 30 Newtons in the positive x-direction is just sufficient to get a 10 kg block moving on a horizontal surface. What s the coefficient of static friction?

Solution: First, find the normal force, N. As in Example 1, .

Just before the block starts moving, the friction force and the applied force are equal and opposite in direction. So Newton s second law gives, in the horizontal direction:

—— ——

5.2. 2 Kinetic Friction

Kinetic friction is the friction force exerted on a moving object. Generally, on a given surface, it is less than the corresponding static friction. It is modeled by an experimentally-derived number, , with the force given by

where again, N is the normal force.

Example 3. The block in example 2, once it s moving, accelerates at a rate of two meters per second per second, or . What s the coefficient of kinetic friction?

Solution: This is just a matter of writing down Newton s second law for the x-direction. As long as forces are purely in the x-direction or y-direction, as here, this can be done easily and accurately. Otherwise, a few adjustments have to be made. The normal force is the same as before. Newton s second law, for the x-direction, is

Put in the mass of 10 kilograms, the acceleration of , and solve for , getting

—

5.3 Statics

In static problems there is no motion, so the acceleration is zero and Newton's second law reads

Systems moving at constant velocity also satisfy this equation. There is nothing special going on, here; these problems are in fact easier than most, because of the lack of acceleration. However, this is an important category of problem, useful in many engineering applications.

Example 4. A block of mass 50 kg hangs from a string. Find the tension in the string.

Solution: The acceleration is zero, so the problem is static. Newton s second law does the rest.

5.4 The Four Forces of Nature

There are four forces in nature: the gravity force, the electromagnetic force, the weak force, and the strong force. The most commonly noticed in everyday life is the gravitational force. Near the surface of the Earth, this is given by

where ***m*** is the mass of the object and . The force will be further studied in a later chapter, including the more general form of the law, which is

———

M and ***m*** are the masses of the two bodies, , ***r*** is distance between the centers of mass of the two bodies, and is a unit vector that points from the source of the gravity field toward the affected particle, or point of interest.

Electromagnetic forces involve something called charge. Light is a manifestation of electromagnetic radiation. There is the electrostatic force law, also called Coulomb's law, given by

Again, this will be studied in great detail later. Notice how similar it is to the gravitation law, though the electromagnetic forces are far stronger.   These forces are common in everyday life, but largely taken for granted. Besides giving us electricity, this force prevents us from falling through the ground into the Earth (that is, it creates the normal force), is responsible for all of chemistry, atomic bonds of every kind. The electric force creates magnetism, enables communication by radio waves, and so forth.

The strong force acts between the different constituents of the nucleus, whereas the weak force is involved in various decay processes, so weak that it doesn't form permanent bound states. The strong force is about 100 times as strong as the electromagnetic force, and about 100,000 times as strong as the weak force. Gravity is extremely weak--about 10^{-39} as strong as the electromagnetic forces.

5.5 Two Body Problems

Many problems involve two bodies, usually held together by a string or some other force. It's important to realize that Newton's equations must be applied to each body independently. It's best to draw a diagram and then put a dotted line around the body you want to consider first, then write down the equation for that body, considering only the forces that are directly applied to that body! Usually, it's necessary to generate two or three equations, and then solve them simultaneously.

Sometimes it's possible to consider two bodies as a single system. In this case, forces between the two bodies are purely internal forces, and don't have to be included in the equation. This can occasionally save a step.

Example 5. Block 1 , mass 10 kg, is attached by a horizontal string to block 2, mass 5 kg. Block 1 is attached to another string and a horizontal force of 100 Newtons is applied. (A) What s the acceleration of the system? (B) What is the tension in the string attached to the second block?

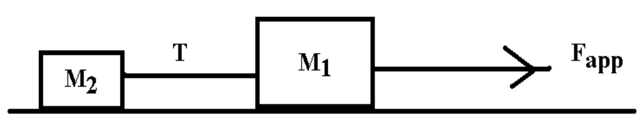


Figure 1: Example 5

Solution: (A) Write down ***F=ma*** for each of the blocks, and solve the equations. Since there is no friction, and the action is all horizontal, we don t have to worry about the vertical forces (the normal and gravitational forces).

These are the three equations we need to solve for the three unknowns, T, a_1 and a_2. The first two are from Newton s second law, and the last from the simple observation that the two blocks will have the same acceleration. We will call this acceleration simply a and substitute it in for a_1 and a_2.

The system has been reduced down to two equations and two unknowns. We can eliminate T by adding the two equations.

(B) Now we can substitute into the equation for the second block and easily find the tension in the string attached to the second block.

Notice that this tension is smaller than the tension in the string drawing the whole system, which would be 100 Newtons.

5.6 Drag Forces

Drag forces result from atoms and molecules brushing against an object, and so they are similar to friction forces. Drag forces are usually connected with gases and fluids. For an object falling through air or some other gas, the drag force, for low velocities and 'laminar flow', is given by

In 'laminar Flow' , the fluid essentially separates into layers around the body, and the friction between these layers creates the drag. The minus sign means the drag force always points opposite the direction of motion. The constant depends on the shape of the object and the thickness and stickiness of the fluid--a property called the ***viscosity*** of the fluid, which is found experimentally. Since the drag force increases with speed, there will be a ***terminal velocity*** at which there is no further net acceleration, nor change in speed. Since Newton's law is

then when ***a = 0***, it follows that terminal velocity is given by

For more rapidly falling objects, or for large bodies at moderate speeds, a turbulent wake forms behind the body, reducing the pressure behind compared to in front. The drag force is then given by

Here C_D is taken to be a constant, ***A*** the cross sectional area, and is the density, while is just a catch-all constant which combines all the other constants into one for convenience.

You have to be careful of the minus sign, since unlike the previous equation, this expression is always positive, so the user must supply a sign. When up is defined as the positive direction, the sign should be negative if the object is going up, positive if it is going down. It's important to realize, however, that in reality C_D depends on the velocity, generally increasing with speed.

Again, a terminal velocity can be found by setting ***a = 0***:

$$\sqrt{\frac{\;}{\;}}$$

Dependence on means equations involving this kind of drag may not have exact solutions. When the gravity acceleration is constant, however, it is, indeed, possible to find an exact solution, just as in the case where the drag depends on v. It's not easy, of course, but is just a lot of calculus. Since this is a little beyond typical first year physics, the solution of this problem is left for the examples.

5.7 Problem-Solving Technique

Most of the problems involving Newton's laws go from hard to nearly impossible. It's important to have good techique, a good set of moves to make. Make the same moves every time, never try to go on shortcuts. In this way, you learn good technique from the easy problems, which will help you on the hard questions. A suggested series of steps is the following:

1. READ the question a couple times.
2. DRAW a picture, while rereading the question.
3. NAME the force vectors, and all other quantities, with letters reminding you what they are.

4. DRAW the force vectors in the picture.
5. WRITE down (aka the 'F=ma equation').
6. Find an expression for each force in terms of components (usually x and y components)
7. Load the expressions into the F=ma equation, and rip out all the x-components, making one equation, and all y-components, getting a second equation.
8. Solve these two equations simultaneously, using only letters, not numbers.
9. Plug in the numbers.

Step 8 says it's important to carry the letters through the calculation. First of all, when you reach the bottom line you will have a general relationship, which can be very useful. Second, you will be less likely to make a mistake. Combine a bunch of numbers, and you've hidden the algebraic dynamics! So it's tougher to see or find your errors. Now let's go to a few example problems, to show how we can apply Newton's fabulously valuable law.

5.8 More Examples

5.8.1 Weight and the Normal Force

Example 6. Elevator Weight. Suppose your mass is 70 kilograms. (a) What is your apparent weight in an elevator (on Earth) which is accelerating downward at 5 m/s^2 ? (c) When it's braking at 5 m/s^2 ?

Solution: (A) Gravity and the normal force act on you inside the elevator, nothing else. If you're standing on bathroom scales, the number on the scales corresponds to the normal force (being an 'equal and opposite reaction' to the normal force). For part A, downward acceleration is negative, so $a = -5\ m/s^2$ .Then:

(B) Same problem, but with a positive acceleration (braking a downwards fall, or negative velocity, requires and upwards acceleration):.

From bantamweight to Sumo wrestler, all in the same elevator.

Example 7. Block on Plane. A 50 kg block rests on a plane inclined at 30 degrees with respect to the horizontal. The plane is frictionless. (A) What is the acceleration of the block? (B) What is the normal force?

Solution: See the figure. Though not strictly necessary, using a coordinate system where the y-direction is perpendicular to the incline and the x-coordinate is parallel to the incline can make such problems easier. We have from Newton's 2nd law:

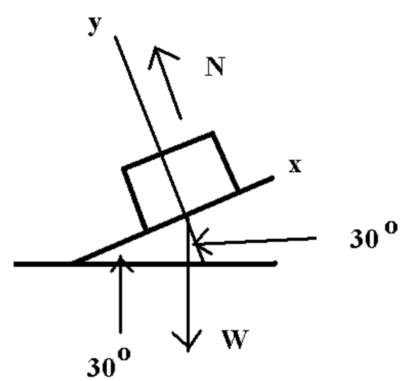


Figure 2: Example 7

where . From these two equations, obtain

Example 8. Rope Force. Find the normal force on a 200 kg block being pulled by a rope inclined at thirty degrees with respect to the horizontal. Assume the applied force is 1000 Newtons.

Solution: The upward component of the applied force will relieve some of the downward pressure, which means the reaction force (the normal force) will also be smaller.

We're just interesting in the y-component, which is:

Plug in the numbers.

—

5.8.2 Friction Forces

Example 9. A 50 kg block is on a horizontal surface with . What minimum force must be exceeded in order to start it moving?

Solution: Let be the applied force. Then for movement to occur,

Example 10. A block massing 30 kg is shoved at an initial speed of 10 m/s across a rough horizontal surface with coefficient of kinetic friction 0.25. (A) Find the friction force (B) What s the acceleration? (C) How far does it travel before coming to rest?

Solution: (A) The friction force is given by the coefficient of kinetic friction times the normal force:

(B) Newton s second law then gives

(C) To find out how far it travels before coming to rest, we can use the ballistics equation,

The final velocity, **v**, is zero, and the initial velocity, $\mathbf{v_o}$, is 10 m/s. Take the initial position to be . Then

Example 11. A block rests on an incline having static friction given by .
What is the maximum angle the incline can be without allowing the block to move?

Solution: See the previous incline problem., number 7, for a picture of this situation. The problem can be solved in three steps. 1. Find N, the normal force. 2. Use N to get the force of static friction. 3. The sum of this force and that of the gravity force acting down the incline is zero. The normal force is given by , as before. The part of the gravity force acting down the incline is given by , as is clear from the diagram.

Newton s law, for a static situation gives us

If the sum of those forces is greater or equal to zero, the block won t slide down the incline. Divide both sides by , getting

Example 12. A 50 kg block rests on a surface with kinetic friction coefficient of 0.2. What is the acceleration of the block if pulled by a horizontal force of 200 Newtons?

Solution: This is a one-dimensional problem.

Example 13. Another block. A 50 kg block on a surface with kinetic friction 0.3 is pulled by a 400 N. force directed at a 30 degree angle with respect to the horizontal. (A) What is the normal force? (B) What is the acceleration of the block?

Solution: (A) The normal force for a similar situation has already been found. It will be

—

(B) Now that we have the normal force, we can find the acceleration in the x-direction.

5.8.3 Statics

Example 14. A 50-kg kid on a one-rope swing is suspended at a thirty-degree angle with respect to the vertical by a second horizontal rope, attached to the kid and going off in the positive x-direction. Find the tensions in the ropes.

Solution: Newton's second law reads:

where $\boldsymbol{T_1}$ is the rope attached overhead, and $\boldsymbol{T_2}$ the horizontal rope. $\boldsymbol{T_1}$ makes a 120° angle with respect to the positive x-axis. Fill in the forces:

Figure 3: Kid and Two Ropes

Recall that vectors can be formed by taking their magnitudes and multiplying by a unit vector, and in the plane, that unit vector can be written , where is the angle measured from the positive x-axis.

The x-component reads

while the y-component reads

From the y-equation,

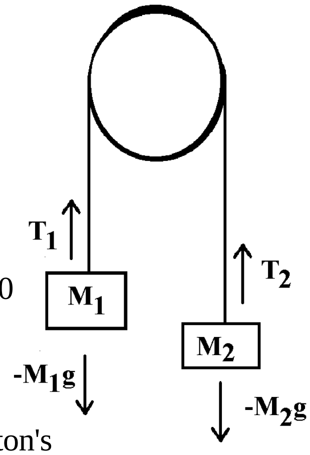
From the x-equation,

Example 15. A 10 kg picture hangs on the wall, supported by two strings attached to a point centered above the picture. The strings are attached to the corners of the picture, making an equilateral triangle. Find the tension in the strings.

Solution: By symmetry, the tensions are the same in each string, and the x-components of the forces are equal and opposite. The y-components give

5.8.4 2-Body Problems

Example 16. Atwood's machine. Two masses, M1 and M2, are connected by a light
string over an essentially massless, frictionless pulley. Let M1=20 kg, M2=40 kg. (A) Find the
acceleration of the system; (B) Find the tension in the string.



Solution: Newton's equations are:

The tensions, in this case, are identically equal, . In addition, since one block accelerates up and the other down, . Substituting this nto the first equation and multiplying both sides by -1 gives

Add this equation to the second equation. This results in

Solving for a_2 results in

To find the tension in the string, substitute this into Newton s equation for the second mass and solve for T:

Now solve this equation for the tension, T.

Example 17. Two blocks, M_1 and M_2 , are connected by a string and pulley as shown, M_1 on a frictionless table, and the other, M_2 dangling off the edge. The pulley is assumed to be frictionless and massless, so it doesn't affect the motion. M_1 has mass 20 kg, while h M_2 as mass 40 kg. (A) What is the acceleration of the system if the table is frictionless? (B) Has coefficient of kinetic friction ?

Solution: (A) Define the positive direction as going around the pulley from left to right. The total mass is $M_1 + M_2$. Only the gravitational force on M_2 acts to accelerate this system, so, treating the two blocks as one, we have:

(B) Adding friction just means there is one more force that s slowing the system down. The equations are the same as before, but with an added friction term:

5.8.5 Drag Forces

Example 18. Suppose at a given instant of time, the acceleration of a 0.5 kg object falling through a liquid is -2 m/s^2 while the velocity is -3 m/s. (A) Find the viscosity, , of the liquid. (B) Find the terminal velocity.

Solution: Use Newton s second law.

The terminal velocity is achieved when the acceleration, a, is zero, so: