Chapter 4

NEWTON'S LAWS OF MOTION

This chapter will present Newton's laws of motion. By far the most important is the second law, especially since the first and third laws can be obtained from it.

4.1 Newton's Laws

4.1.1 The Three Laws

Newton's three laws of motion are

1. Bodies move at constant velocity unless acted on by a force.
2. The change in the momentum of a body equals the vector sum of all external forces acting on that body.
3. For every applied force there is an equal and oppositely directed counterforce on the agent applying the original force..

First Law: This is common sense. Things don't usually speed up or slow down by themselves. Something must act, a force must cause the change. This law actually follows from the second law.

Second law: This is probably the most important and useful equation in all of physics. It is usually written as but originally was written in terms of a change of momentum, $p = mv$, with time. For our purposes, however, it will be

$$m\vec{a} = \sum_{i=1}^{n} \vec{F}_i \tag{4.1}$$

On the right hand side, the F_i are **external** forces. This is an important distinction, and just means that objects cannot change their state of motion by exerting a force on themselves–in other words, you can't fly by pulling on your bootstraps! It might be thought that rockets violate this, but actually, the hot gasses are pushing the ship.

Third Law: This at first may seem strange: when you push on something, it pushes back! We usually don't notice this because we unconsciously brace ourselves. Stand flat-footed next to a wall and push, and of course you fall backwards.

Notice that the first law is a consequence of the second law:

$$m\vec{a} = \sum F_i = 0 \rightarrow m\frac{d\vec{v}}{dt} = 0 \rightarrow \vec{v} = constant$$

This equation says exactly what the first law says, but with math instead of words: no force, constant velocity.

The third law can also be understood in terms of the second law, but it's somewhat more complicated, requiring the notion of center of mass coordinates.

4.1.2 Center of Mass Coordinates

Center of mass coordinates are defined by

$$M_{tot}\vec{r}_{cm} = \sum_{i=1}^{n} M_i\vec{r}_i \tag{4.2}$$

where

$$M_{tot} = M_1 + M_2 + ... + M_n \tag{4.3}$$

and $\vec{r}_i$ is the position vector pointing to the mass M_i. $\vec{r}_{cm}$ points to the center of mass. By taking two derivatives of this equation with respect to time, we obtain the velocity of the center of mass, $\vec{v}_{cm}$, and the acceleration of the center of mass, $\vec{a}_{cm}$:

$$M_{tot}\,\vec{v}_{cm} = \sum_{i=1}^{n} M_i \vec{v}_i \tag{4.4}$$

$$M_{tot}\,\vec{a}_{cm} = \sum_{i=1}^{n} M_i \vec{a}_i \tag{4.5}$$

Defining the center of mass coordinates and then writing down equations in terms of relative displacement from the center of mass is useful for complicated systems involving several particles. This is particularly true in particle physics. Here, our main purpose is to use the idea to show that Newton's third law follows from the second law.

For every action there is an equal and opposite reaction implies there are two bodies, one acting on the other. Call the masses of the two bodies M_1 and M_2. Suppose M_1 acts on M_2 by exerting a force, F_2. Then M_2 may, or may not, exert a force on M_1. We designate this unknown reaction force by F_x, and make no other assumptions about it. We'd like to show that $F_x = -F_2$. Newton's second law for the two individual masses gives the following two equations:

$$M_1 \vec{a}_1 = \vec{F}_x$$

$$M_2 \vec{a}_2 = \vec{F}_2$$

We generate another equation using the concept of center of mass. This time, we view the two masses as a single system. The force F_2 and possible force F_x are internal forces– internal to the combined system– and so they will not appear in Newton's law for this system, which in center of mass coordinates reads

$$M_{tot}\vec{a}_{cm} = M_1\vec{a}_1 + M_2\vec{a}_2 = \sum \vec{F}_i = 0$$

This means that

$$M_1\vec{a}_1 = -M_2\vec{a}_2 \rightarrow \vec{F}_x = -\vec{F}_2$$

So Newton's third law is really a consequence of the second law.

4.2 Problem-solving Technique

Easy problems require only a cursory examination of Newton's three laws. Most involve simple applications of $F = ma$. Typical steps are:

1. Write down the F=ma equation.
2. Plug in and solve for the unknown quantity

4.3 Examples

4.3.1 Newton's Laws

Example 1. The Box and the Rhino. A box with mass 40 kilograms rests on a frictionless surface. A rhinoceros pushes it with a horizontal force of 200 Newtons. (A) What's the acceleration? (B) Same problem, with friction a friction force of 100 Newtons acting in the opposite direction.

Solution: This can be solved with Newton's second law. Let m_b be the mass of the box, a the acceleration. (A)

$$m_b a = F_{rhino} \rightarrow 40a = 200 \rightarrow a = 5\ m/s^2$$

(B) More on friction will be given in the next chapter. Here, it's easy:

$$m_b a = F_{rhino} - F_{friction} \rightarrow 40a = 200 - 100 = 100 \rightarrow a = 2.5\ m/s^2$$

Example 2. Flying Leap. You leap out of an airplane, skydiving towards the ground below, reaching a terminal velocity of 120 miles per hour. If your mass is 70 kg, what is the magnitude of the air drag force?

Solution: This is, in fact, very similar to part (B) of the previous problem. Let F_{drag} be the drag force. 'Terminal velocity' is a constant velocity that results when the upward force of air friction, at sufficiently high speed, exactly cancels the downward force of gravity. Then the net acceleration is zero:

$$ma = F_{grav} + F_{drag} = 0 \rightarrow F_{drag} = -F_{grav} = -(-mg) =$$

$$= mg = 70 \cdot 9.8 = 686 \ Newtons$$

Example 3. Braking Car. A 2,000 kg car traveling 100 m/s manages to brake uniformly to 50 m/s in four seconds. What is the magnitude of the force slowing it down , assuming it is constant? Note: Physicists rarely use the word deceleration acceleration can be positive or negative, depending on direction, and that covers it.

Solution: Use Newton's second law. Let m be the mass of the car, a its acceleration, F the braking force. First, find the acceleration.

$$a = \frac{v_f - v_i}{\Delta t} = \frac{50 - 100}{4} = \frac{-50}{4} = -12.5 \ m/s^2$$

$$F = ma = 2000 \cdot (-12.5) = -25,000 \ N$$

Example 4. Speedboat Drag. A speedboat's engine is capable of applying a force of 3,500 Newtons. If the 1,000 kg boat accelerates through the water at 2 meters per second squared, what is the drag force?

Solution: This is actually very similar to the previous problem, which involved a friction force. Let m be the mass of the boat, a the acceleration, F_E the force of the engines, F_D the drag force,. Then:

$$ma = F_E + F_D \rightarrow F_D = ma - F_E = 2 \cdot 1000 - 3,500 = -1,500 \ N$$

Example 5. Hippo vs. Rhino. A hippo and rhinocerous have a tug of war, battling over a 2 metric ton box of delectable vegetables. If the hippo pulls with a force of 1,000 N. in one direction and the rhino pulls with a force of 800 N. in the other direction, what is the net acceleration of the box?

Solution: Another easy problem. m=mass of box, F_r=rhino force, F_h=hippo force. Let the hippo force be in the positive x-direction.

$$ma = F_h - F_r \rightarrow 2000a = 1000 - 800 = 200 \rightarrow a = 0.1\ m/s^2$$

Example 6: Space Shuttle. What is the acceleration of the space shuttle after liftoff? DATA. Masses: orbiter: 75,000 kg; payload: 29,500 kg ; tank: 35,400 kg empty; fuel: 616,500 kg oxygen and 102,000 kg hydrogen; boosters: 590,000 kg each. Thrust: boosters 11,800,000 Newtons each at launch; space shuttle main engine: 1,700,000 Newtons each (there are 3 of them).

Solution: The two forces are the thrust of the rocket engines and the force of gravity. The total mass of the rocket at launch is 858,400 kg. We have

$$ma = \sum F = F_{thrust} + F_{gravity} = F_thrust - mg$$

Add up all the thrusts, getting

$$F_{thrust} = 2 \cdot 11,700,000 + 3 \cdot 1,700,000 = 28,700,000\ Newtons$$

Next, plug in the numbers.

$$858,400a = 28,700,000 - 858,400 \cdot 9.8 \rightarrow a = 23.63\ m/s^2$$

Example 7. The Donkey and the Giant Robotic Beetle. A block with mass of 2,000 kilograms rests on a frictionless surface. Two ropes are attached. One is pulled horizontally north by a donkey with a force of 400 N. while the other is pulled horizontally in a direction thirty degrees north of east by a giant robotic beetle, with a force of 800 N. (A) Find the x and y components of the acceleration. (B) What is the magnitude and direction of the resultant force? (B) What is the acceleration of the block? (C) Starting from rest, how fast is it going after five seconds?

Solution: $\vec{F}_d$ =force of donkey; $\vec{F}_b$ = force of robotic beetle; m = mass of block. Let North be the +y-direction, East the +x-direction. This problem requires the vector form of Newton's law in two dimensions. The surface is assumed frictionless.

$$m\vec{a} = \vec{F}_d + \vec{F}_b = 400(\cos(90), \sin(90)) + 800(\cos(30), \sin(30))$$

$$2000(a_x, a_y) = (0, 400) + 800\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$2000(a_x, a_y) = \left(800 \cdot \frac{\sqrt{3}}{2}, 800 \cdot \frac{1}{2} + 400\right) = (400\sqrt{3}, 800)$$

(A) The right-hand side of the above equation is the resultant force, in vector form. To find the magnitude and direction, use simple right triangle math:

$$||\vec{F}|| = \left(F_x^2 + F_y^2\right)^{1/2} = \left((400\sqrt{3})^2 + 800^2\right)^{1/2} =$$

$$= (480,000 + 640,000)^{1/2} = (1,120,000)^{1/2} = 1,058.3\ N$$

For the direction, use the tangent:

$$\tan\theta = \frac{F_y}{F_x} = \frac{800}{400\sqrt{3}} = 1.155 \rightarrow \theta = \tan^{-1} 0.866 = 49.1^o$$

(B) It's easy to get the x- and y-components of the acceleration. Rip out the x and y- components out of Newton's second law.

$$2000a_x = 400\sqrt{3} \rightarrow a_x = 0.346\ m/s^2$$

$$2000a_y = 800 \rightarrow a_y = 0.4\ m/s^2$$

(C) Since the forces are constant, so is the acceleration, so $v = at$. Just multiply the magnitude of the acceleration by the time:

$$||\vec{a}|| = \left(a_x^2 + a_y^2\right)^{1/2} = \left(0.346^2 + 0.4^2\right)^{1/2} = 0.529\ m/s^2$$

$$v = at = 0.529 \cdot 5 = 2.64\ m/s$$

Example 8. Skydiving. Suppose a 70 kg skydiver, at 10,000 meters and falling at 30 m/s, experiences a constant drag force of 750 N.
(A) What is the net force? (B) What is the acceleration?

Solution: (A) There are two forces: the drag force and gravity force (and a negligible buoyancy force).

$$F_{net} = F_{drag} + F_{grav} = 750 - mg = 750 - 70 \cdot 9.8 = 64 \; N.$$

Notice that the drag force is positive, acting upwards.

(B) Newton's second law gives the acceleration:

$$ma = F_{net} = 64 \Rightarrow a = 64/m = 64/70 = 0.914 \; m/s^2$$

Even though the skydiver is falling down, with negative velocity, the acceleration is upwards– because the skydiver is slowing down.

Example 9. Elevator (A) What force must the cable of an elevator massing 2000 kg, including load, exert if the elevator is to rise at a steady speed? (B) Same question, except the elevator is accelerating upwards at $2m/s^2$.

Solution: There are two forces, the cable force, called T for the tension in the cable, and the gravity force, mg. Newton's law reads

$$ma = T - mg$$

(A)At steady speed, there is no acceleration, so $a = 0$. Plug this in and solve for T:

$$0 = T - mg \rightarrow T = mg = 2000 \cdot 9.8 = 19,600 \; N.$$

(B) Now the elevator is accelerating upwards, so set $a = 2 \; m/s^2$:

$$ma = T - mg \Rightarrow T = m(a + g) = 2000 \cdot (2 + 9.8) = 23,600 \; N$$

Example 10. Donkeys. Two donkeys pull a barge along a canal. The donkeys are on either side, and each exerts a force of 500 N. directed at a thirty-degree angle away from the canal. What's the drag force, if the barge moves at constant velocity?

Solution: Let the x-axis be oriented along the canal. Then one donkey, say donkey number 1, pulls at an angle of -30^o with respect to the x-axis, while the other pulls at $+30^o$. Steady velocity, so again $a = 0$. Let the donkey forces be F_1 and F_2, the drag force F_{drag}. Then:

$$m\vec{a} = \vec{F}_1 + \vec{F}_2 + \vec{F}_{drag} = \vec{0}$$

$$\rightarrow \vec{F}_{drag} = -\vec{F}_1 - \vec{F}_2$$

The donkey forces are given by their magnitudes times their directions—500 Newtons times unit vectors pointing in the two directions.

$$\vec{F}_1 = 500\,(\cos -30^o, \sin -30^o) = \left(500 \cdot \frac{\sqrt{3}}{2}, 500 \cdot (-\frac{1}{2}\right) =$$

$$= \left(250\sqrt{3}, -250\right)$$

$$\vec{F}_2 = 500\,(\cos 30^o, \sin 30^o) = \left(500 \cdot \frac{\sqrt{3}}{2}, 500 \cdot (\frac{1}{2}\right) =$$

$$= \left(250\sqrt{3}, +250\right)$$

Putting these together in the equation for drag, obtain

$$F_{drag} = \left(-500\sqrt{3}, 0\right)$$

Example 11. Mud-Trucking A truck going 20 meters per second hits a stream. The engine dies immediately, and the jeep slows to a stop in three seconds. If the mass of the truck is 1500 kg, what average force was applied to the truck by the mud and water of the river?

Solution: First find the average acceleration.

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{0 - 20}{3 - 0} = \frac{-20}{3}\ m/s^2$$

Next, plug into the second law:

$$ma = F \rightarrow 1500(-20/3) = F = -10,000\ Newtons$$

There are other forces on the right-hand side, of course–the normal force and gravity force, for example–but here, only the horizontal forces count, in this case the mud and water drag on the truck.

Example 12. Stacking Elephants. One elephant stands on top of another elephant. Each elephant weighs 8,000 Newtons. In each case, each of their four legs takes an equal amount of weight. (A) What is the reaction force of the ground on the bottom elephant? (B) What is the reaction force of the back of the bottom elephant on the top elephant?

Solution:,The total weight,w_{tot}, acting down on the earth is $w_{tot} = 2 \cdot 8,000 = 16,000\ Newtons$. So the reaction force is exactly this same number but acting up instead of down.
(B) The top elephant exerts a downwards force of 8,000 Newtons on the back of the bottom elephant, so the reaction force is also 8,000 Newtons, acting up instead of down.

Example 13: Center of Mass Coordinates. Suppose $M_1 = 4\ kg$ is going to the right at 3 m/s, while $M_2 = 5\ kg$ is going to the left at 5 m/s. If initially, at $t = 0$, M_1 is at $x_1 = 5$ and M_2 is at $x_2 = -2$, find (A) Find the initial center of mass. (B) Find the center of mass at any time. (C) find the velocity of the center of mass.

Solution: Apply the definitions. The total mass is

$$M_{tot} = 4 + 5 = 9\ kg$$

The center of mass is obtained from the center of mass equation:

$$M_{tot}\vec{r}_{cm} = \sum_{i=1}^{n} M_i \vec{r}_i = M_1 x_1 + M_2 x_2$$

$$\vec{r}_{cm}(t) = \frac{4}{9}x_1 + \frac{5}{9}x_2$$

The (t), of course, indicates 'function of time'. Using this equation, the initial center of mass is

$$\vec{r}_{cm}(0) = \frac{4}{9}5 + \frac{5}{9}(-2) = \frac{10}{9}\ meters$$

(B) To find the center of mass at any time, use the fact that the blocks are traveling at constant velocity. Then, use $x = vt + x_0$ for each of them, getting:

$$x_1 = 3t + 5 \qquad x_2 = -5t - 2$$

The center of mass at any time is, therefore:

$$\vec{r}_{cm}(t) = \frac{4}{9}(3t + 5) + \frac{5}{9}(-5t - 2) = -\frac{13}{9}t + \frac{10}{9}$$

(C) The velocity can be obtained by inspecting the previous equation, where it appears in front of the t, as always, for constant velocity.

$$\vec{v}_{cm}(t) = -\frac{13}{9}\ m/s$$