MODERN PHYSICS

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September 24, 2001

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Chapter 1

Special Relativity

Special Relativity was motivated by Einstein's idea that all theories of physics ought to be invariant under Lorentz transformations, just like Maxwell's equations. Newton's laws, therefore, and the usual concepts of momentum and energy, had to be altered so as to be Lorentz invariant.

1.1 Coordinate Transformations

1.1.1 Galilean Transformations

The Galilean tranformations relate measurements of an observer O to those of a second observer, O', who is moving at constant velocity with respect to the first coordinate system. These are:

$$t' = t \tag{1.1}$$

$$x' = x - vt \tag{1.2}$$

$$y' = y \tag{1.3}$$

$$z' = z \tag{1.4}$$

These equations are completely intuitive, and few would challenge them. It can be shown that Newton's laws are invariant under these transformations.

1.1.2 Lorentz Transformations

Einstein (and others) noted that Maxwell's equations were not invariant under Galilean transformations. This means that observers in different states of relative motion would get different answers for the same physical problem, obviously an unsatisfactory state of affairs. The Lorentz Transformations leave Maxwell's equations unchanged. They are

$$t' = \gamma \left(t - \frac{v}{c^2} x \right) \tag{1.5}$$

$$x' = \gamma\left(x - vt\right) \tag{1.6}$$

$$y' = y \tag{1.7}$$

$$z' = z \tag{1.8}$$

1.1.3 The Basis Theorem

The basis theorem relates derivatives in one coordinate system to derivatives in another coordinate system. It's really a chain rule, and will be useful in some of the mathematical proofs. **Basis Theorem** Let $\left(x^0, ..., x^n\right)$ and $\left(y^0, ..., y^n\right)$ be a second coordinate system. Let V represent a differential operator expressed in the $\{y^i\}$ coordinate system. Then in the x-coordinate system,

$$V = \sum_{i=1}^{n} V x^i \frac{\partial}{\partial x^i}$$

Example. Use the basis theorem to convert the Laplacian from Cartesian coordinates in two dimensions to polar coordinates. **Solution:** The coordinate transformations are given by

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

Now apply the basis theorem, setting $V = \partial/\partial x$, then $\partial/\partial y$:

$$\frac{\partial}{\partial x} = \frac{\partial r}{\partial x}\frac{\partial}{\partial r} + \frac{\partial \theta}{\partial x}\partial\partial\theta = \frac{x}{\sqrt{x^2 + y^2}}\frac{\partial}{\partial r} - \frac{y}{\sqrt{x^2 + y^2}}\frac{\partial}{\partial \theta}$$

Now, switching everything to polar:

$$\frac{\partial}{\partial x} = \cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}$$

For the other partial derivative operator, we have:

$$\frac{\partial}{\partial y} = \frac{\partial r}{\partial y}\frac{\partial}{\partial r} + \frac{\partial \theta}{\partial y}\partial\partial\theta = \frac{x}{\sqrt{x^2 + y^2}}\frac{\partial}{\partial r} + \frac{y}{\sqrt{x^2 + y^2}}\frac{\partial}{\partial \theta}$$

Again, switching everything to polar:

$$\frac{\partial}{\partial y} = \sin\theta\frac{\partial}{\partial r} + \frac{\cos\theta}{r}\frac{\partial}{\partial \theta}$$

Next, convert the Laplacian using these operators. This is lengthy but straightforward.

$$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=\left(\cos\theta\frac{\partial}{\partial r}-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)\left(\cos\theta\frac{\partial}{\partial r}-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)+$$
$$+\left(\sin\theta\frac{\partial}{\partial r}+\frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\right)\left(\sin\theta\frac{\partial}{\partial r}+\frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\right)=$$
$$=\cos^2\theta\frac{\partial^2}{\partial r^2}+\frac{\sin^2\theta}{r}\frac{\partial}{\partial r}-\frac{\cos\theta\sin\theta}{r}\frac{\partial^2}{\partial\theta\partial r}+\frac{\cos\theta\sin\theta}{r^2}\frac{\partial}{\partial\theta}-$$
$$-\frac{\cos\theta\sin\theta}{r}\frac{\partial^2}{\partial r\partial\theta}+\frac{\sin\theta\cos\theta}{r^2}\frac{\partial}{\partial\theta}+\frac{\sin^2\theta}{r^2}\frac{\partial^2}{\partial\theta^2}+$$
$$+\sin^2\theta\frac{\partial^2}{\partial r^2}+\frac{\cos^2\theta}{r}\frac{\partial}{\partial r}+\frac{\cos\theta\sin\theta}{r}\frac{\partial^2}{\partial\theta\partial r}-\frac{\sin\theta\cos\theta}{r^2}\frac{\partial}{\partial\theta}+$$
$$+\frac{\sin\theta\cos\theta}{r}\frac{\partial^2}{\partial r\partial\theta}-\frac{\cos\theta\sin\theta}{r^2}\frac{\partial}{\partial\theta}+\frac{\cos^2\theta}{r^2}\frac{\partial^2}{\partial\theta^2}=$$
$$=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

1.2 Vectors and Tensors

1.3 Examples

Example 1. A Mercedes sports car passes a hitchhiker at 40 m/s. Five seconds later, the hitchhiker spots a policecar approaching from behind at 50 m/s, 200 meters away. Find the position and speed of the policecar as determined by the driver of the Mercedes at this time.

Solution: Apply the Galilean transformations. Let the primed coordinate system travel with the Mercedes, the unprimed being that of the hitchhiker. Then

$$x' = x - vt = -200 - 40\cdot 8 = -520\ m$$
$$u_x{}' = u_x - v = 50 - 40 = 10$$

Example 2. A beam of protons is traveling at 0.99c around a cyclotron as measured by an observer at rest with respect to the Earth, while a beam of antiprotons travels at the same speed but the opposite direction. What is the speed of the protons as measured by an antiproton?

Solution: Let x' denote the antiproton frame, with x the lab frame. Plugging into the velocity tranformation equation yields

$$u_x' = u_x - v = 0.99c - (-0.99c) = 1.98c$$

So superluminal velocities are possible in old style Galilean physics!

Example 3 An infantryman observers a British Wellington Bomber passing overhead at 100 m/s, headed due East. One minute later, a Nazi warplane passes one kilometer to the south, traveling northeast at 120 m/s. Find the (A)position and (B) velocity at this time, as observed from the Wellington. (C) What is the relative speed?

Solution: Grind with Galileo. Let the primed coordinates be those of the bomber, unprimed those of the infantryman. Let north be the positive y-direction, east the positive x-direction.

$$x' = x - vt = 0 - 100\ m/s \cdot 60\ s = -6000\ m$$

$$y' = y = -1000\ m$$

$$u_x = 120 \cos 45^o = 60\sqrt{2}\ m/s$$

$$u_y = 120 \sin 45^o = 60\sqrt{2}\ m/s$$

$$u_x{}' = u_x - v = 60\sqrt{2} - 100$$

$$u_y{}' = u_y = 60\sqrt{2}\ m/s$$

Example 5: Show that the scalar wave equation is not invariant under Galilean transformations.

Solution: We use the basis theorem to transform the derivative operators. **Example 6.** Show that the Lorentz metric is invariant under the Lorentz transformation.

Example 7. An observer O measures a flash of lightning as located at $t = 1 \times 10^{-3}\ s$, $x = 200\ km$, $y = 50\ km$, and $z = 2\ km$. A second observer, O', is taveling at $0.6c$ relative O. What does O' measure for the coordiantes of this event?

Solution: First, calculate the γ-factor:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.36}} = 1.25$$

Next, apply the Lorentz transformations.

$$x' = \gamma(x - vt) = 1.25(200,000 - 0.6c \cdot 10^{-3}) = 25,000\ m$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = 1.25\left(10^{-3} - \frac{0.6c}{c^2} \cdot 200,000\right) = 7.5 \times 10^{-4}\ s$$

1.4 Postulates of SR

1. Nothing can travel faster than the speed of light, c.
2. Physics is the same when viewed in all inertial frames of reference.

These two postulates are at the foundation of Einstein's theory. The constancy of the speed of light, together with the conditions that transformations from one coordinate system to the moving coordinate system is linear in the coordinates, and that the resulting transformations reduce to the Galilean transformations when relative velocities are small, are sufficient to determine the Lorentz transformations.

Special relativity, it turns out, means measurements of length and time are dependent on the observer, a bizarre but well-tested fact.

1.5 Length Contraction

Consider a meter stick approaching a laboratory observer, O, at some uniform speed v. An observer traveling with the meter stick, O', will naturally measure it to have one meter of length. The lab observer will have a different opinion. O measures the position of both ends, at x_a and x_b, at the same time. Then, taking differences of equation **??**,

$$\Delta x' = \gamma \left(\Delta x - \frac{v}{c^2} \Delta t \right) \tag{1.9}$$

$\Delta x' = {x_b}' - {x_a}'$ is just the proper length, called L_0, while $L = x_b - x_a = \Delta x$ is the length measured by O. Since $t_b = t_a$, we have

$$L = \frac{L_0}{\gamma} = L_0 \sqrt{1 - \frac{v^2}{c^2}} < L_0 \tag{1.10}$$

From this, it appears that as observed by O, the moving rod is contracted. This effect is called Lorentz-Fitzgerald Contraction, after Lorentz and Fitzgerald, who proposed such contraction to explain the results of the Michelson-Morley experiiment.

1.6 Time Dilation

The phenomenon of time dilation is perhaps the most famous of the quirky predictions of Einstein's theory. This, like length contraction, is a real effect, and in fact has been verified by countless experiments, from millions of daily events in particle accelerators, to the rain of cosmic ray-created muons, and even from measurements taken in jets traveling around the world. Moving clocks run slow.

Again, a time difference is written down, using the Lorentz transformation.

$$t_b{}' - t_a{}' = \gamma\left((t_b - t_a) - \frac{v}{c^2}(x_b - x_a)\right) \tag{1.11}$$

Here, as before, O' is moving with at v with respect to O. O measures the interval between two events which, according to O, take place in the same spatial location. Hence $x_b - x_a = 0$, and

$$\Delta t_{moving} = \gamma \Delta t_{proper} \tag{1.12}$$

$\Delta t_{proper}$is the time measured by the observer at rest with respect to the clock. The moving observer will measure a greater length of time between the two events. Since $\gamma \geq 1$, the time measured by the moving observer, O', will be longer than that measured by O, where the clock is at rest. The time has been *dilated*, or enlarged, stretched out to a larger size.

This leads to a famous paradox, called the Twin paradox. Peter and Earl are identical twins. Peter takes off in a spaceship, moving at an appreciable fraction of c, while Earl remains on Earth. The clock carried in his ship, since it is moving, runs slow, so when Peter returns years later he finds Earl has turned into an old man, while Peter is still young.

Many people erroneously think that this discrepancy in age is the paradox, but the real paradox is this: from Peter's point of view, the Earth is traveling away from him at v. Hence Earl is moving, and when Peter returns, it is Earl who should be younger.

This paradox was resolved by noting that Peter actually changes reference frames by accelerating up to speed, then turning around and coming back, again undergoing accelerations. Earl, meanwhile, never changed frames. So the situation isn't truly symmetric. Peter did the moving, so his clock ran slow, while Earl's ran at the normal Earth rate. This is the standard resolution, and it isn't entirely satisfactory, since it would be nice to have a clear-cut calculation that shows Peter's clock slowing down, while Earl's clock jackrabbits along.

1.7 Relativistic Forces

There are a couple of different approaches to defining a relativistic force. Both will be explored here. The most obvious way is

$$\frac{d}{dt} m u_a = F_a \tag{1.13}$$

Here, F_a are the components of the usual force, for $a = 1, 2, 3$, the spatial components only. Multiplying both sides by γ gives

$$\frac{d}{d\tau} m u_a = \gamma F_a \tag{1.14}$$

There remains only the necessity of finding the time component. Take the inner product of both sides with u^a:

$$u^a \frac{d}{d\tau} m u_a = \gamma F_a u^a$$

Use the fact that

$$u^a \frac{d}{d\tau} = \frac{1}{2}\frac{d}{d\tau}u^a u_a = \frac{1}{2}\frac{d}{d\tau}(-c^2) = 0$$

to conclude that

$$F_a u^a = 0 \to F_0 u^0 = -F_i u^i$$

1.8 Relativistic Momentum and Energy

1.9 The Compton Effect

1.10 Examples

1.10.1 Postulates of SR

1.10.2 Length Contraction

1.10.3 Time Dilation

Example: Trip to α-Centauri
Peter climbs aboard his starship and travels to α-Centauri, 4.1 light years away, traveling at 0.8c relative Earth. When he returns, how much younger will he be than his twin brother Earl, who remained on Earth?

Solution: Finding how much Earl has aged is simple, since it uses pre-relativity.

$$d = vt \to t_{tot} = \frac{4.1}{0.8} \times 2 = 10.25\ yrs$$

Meanwhile, the clock in Peter's cabin will register a proper time of

$$\frac{\Delta t_{proper}}{\sqrt{1 - v^2/c^2}} = \Delta t_{earth} = 10.25\ yrs \to \Delta t_{proper} = 10.25\sqrt{1 - v^2/c^2} =$$

$$= 10.25\sqrt{1 - 0.8^2} = 10.25 \cdot 0.6 = 6.15\ yrs$$

So Earl will be 4.1 years older than Peter.

Example: Muons from Space
A balloon at 2,600 meters of elevation measures a flux of 458 muons per second. At sea level, the flux is only 87 muons per second. What's the average speed of the muons, given that their normal half life is $1.87 \times 10^{-6}\ s$?

1.10.4 Relativistic Forces

1.10.5 Relativistic Momentum and Energy

1.10.6 Relativistic Forces

1.10.7 The Compton Effect

Chapter 2

General Relativity

Einstein is most famous for the equation $E = mc^2$, but his greatest contribution is probably the theory of General Relativity, which describes gravitation in terms of the curvature of spacetime.

Chapter 3

Pre-quantum Mechanics

In this chapter, we examine some of the ideas leading up to quantum mechanics. The most important of these is the Bohr model, which was a major breakthrough in the understanding of the light given off by hot gases. Later, De Broglie came up with his theory of matter waves, and Heisenberg invented his uncertainty principle.

3.1 Planck's Radiation Law

3.2 Photoelectric Effect

3.3 The Bohr Model

Neils Bohr, with a combination of luck and insight, assembled a classical theory with quantum ansatz that resulted in a description of the electronic energy levels in hydrogen and other elements. This simple model also led to an understanding of the periodic table of the elements.

To create the Bohr Model, follow these steps. First, write down the classical force equation for an electron in circular orbit around a proton.

$$-\frac{mv^2}{r} = -\frac{ke^2}{r^2} \tag{3.1}$$

Second, make the quantum leap, which is

$$L = pr = n\hbar \tag{3.2}$$

There is no justification for this except that it works. Solve this for v:

$$v = \frac{n\hbar}{mr} \tag{3.3}$$

and substitute it back into the first equation, getting, after a little algebra,

$$r_n = \frac{n^2\hbar^2}{mke^2} \tag{3.4}$$

Here, the r_n indicates that the radius depends on n, which is taken to be a positive integer. The possible orbital radii are quantized, which means they can take on only a countable number of discrete values, countable meaning infinite, but in one-to-one correspondence with the integers. Next, the quantized energies can be found. Start with the energy equation:

$$E = \frac{1}{2}mv^2 - \frac{ke^2}{r} \tag{3.5}$$

Solve equation 3.1 for v and substitute into equation 3.5:

$$E = -\frac{ke^2}{2r} \tag{3.6}$$

Finally, substitute equation 3.4 for r in equation 3.6, and simplify, getting

$$E_n = -\frac{mk^2e^4}{2n^2\hbar^2} \tag{3.7}$$

Often, several of the constants in the above equation are replaced by the single quantity α, given be

$$\alpha = \frac{ke^2}{\hbar c} = 137 \tag{3.8}$$

The quantity α is called the **fine structure constant**, a dimensionless number that is of importance in quantum electrodynamics, in perturbation theory. The special case of $n = 1$, which corresponds to the lowest energy level in the hydrogen atom. The **Bohr radius**, a_0, is given by equation 3.4:

$$a_0 = r_1 = \frac{\hbar^2}{mke^2} = 5.29 \times 10^{-11}\ m. \tag{3.9}$$

This is the approximate radius of the hydrogen atom. The energy for $n = 1$ is:

$$E_1 = -\frac{mk^2e^4}{2\hbar^2} = -2.178 \times 10^{-18}\ J = -13.6\ eV \tag{3.10}$$

The energy levels can then be conveniently written as

$$E_n = -\frac{13.6\ eV}{n^2} \tag{3.11}$$

Bohr's motivation for creating this model, of course, was to explain the spectrum of light given off by hot hydrogen. Many of the lines could now by understood as transitions of electrons from higher to lower energy states, always accompanied with the emission of a photon. This can be written as

$$\Delta E_{nj} = h\nu = \frac{hc}{\lambda} = -13.6\ eV\left(\frac{1}{n^2} - \frac{1}{j^2}\right) \tag{3.12}$$

Rearranging this equation gives the famous Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{j^2}\right) \tag{3.13}$$

where $R = 1.09737 \times 10^{-3} A^{-1}$. This had been found empirically by Rydberg in the previous century.

3.3.1 Corrections to the Bohr Model

There are various modifications of Bohr's model to make it more accurate and generally applicable. First of all, the electron in the hydrogen atom is much lighter than the proton, but the fact that the proton moves a little should also be taken into account. This can be effected by replacing the electron mass m by the *reduced mass* μ:

$$\mu = \frac{m_e m_p}{m_e + m_p} \tag{3.14}$$

This differs only a little from the electron mass, but gives more accurate results. Second, it would be of interest to apply this equation to atoms that are completely ionized but for one electron. In this case, the modification consists of replacing the charge e of the proton, wherever it occurs, with Ze, where Z is the number of protons in the nucleus. This results in factors of Z^2 in the equations, taking into account the higher nuclear charge.

3.3.2 Alternate Method.

Another way of creating a Bohr model is as follows.

1. Write down an expression for the energy
2. Make the quantum leap, $pr = n\hbar$, substituting for the momentum, p
3. Find r where the energy in minimized.

Example. Hot hydrogen. Calculate the wavelength

3.4 De Broglie Waves

3.5 Heisenberg Uncertainty Principle

Chapter 4

Quantum Mechanics

Quantum Mechanics was invented in the early 1920's by Werner Heisenberg and Erwin Schrodinger, using different approaches. No one understands why it appears to work so well–the motivation was to understand the light given off by atoms, such as hydrogen, when it was heated. Heisenberg's matrix mechanics and Schrodinger's wave equation were shown by Heisenberg to be equivalent, however the wave mechanical method of Schrodinger was vastly more popular, due to its intuitive appeal and ease of use.

4.1 The Four Postulates of Quantum Mechanics

4.1.1 Observables and Operators

Postulate 1. For anything that can be observed, called A, there is a mathematical operator $\hat{A}$, with the eigenfunction equation

$$\hat{A}\psi = a\psi$$

yielding eigenvalues a that are the same as the actual observed values.

This is an amazing statement, part of the quantum faith. In the above equation, ψ is called the eigenfunction, and the value a is the eigenvalue corresponding the the eigenfunction. For a given observable, like Energy, Momentum, and angular momentum, there will be a number of eigenvalues and eigenfunctions, usually an infinite number. The "spectrum" of eigenvalues, however, is usually discrete–meaning only certain, separate values are obtained, rather than a continuum. This is the difference between, say, the integers and the real line. The integers form a discrete set of numbers, just like the different energy levels in an atom.

4.1.2 Quantum Measurement

Postulate 2. The measurement of an observable A, yielding an answer a, in a quantum system leaves the system in the state described by the wavefunction

ψ_a, which has eigenvalue a.

This is another article of faith. Quantum mechanical systems render probabilities rather than certainties. However, once a system is measured as being in a state ψ_a, it is assumed to evolve subsequently, with ψ_a as a starting point. If this were not the case, all predictive power–even the inexact predication of future probabilities, would be impossible,

This postulate is the famous "Collapse of the Wave Function". Systems are thought to have no definite energy or momentum, only probable energies and momenta, until someone disturbs the system by looking at it. Then the system collapses into a particular state, and renders a value to the observer.

4.1.3 State Functions and Expectation Values

4.1.4 Time Development of the State Function

4.1.5 Born's Probability Waves

4.2 Schrodinger's Equation

The first and most important postulate of quantum mechanics is that the Schrodinger Wave equation gives solutions that are physically meaningful. This must be taken on faith, and on the basis of experimental evidence, since the derivation of Schrodinger's equation is as mysterious as is its incredible success. We start with the classical energy, multiplied by a wave function, Ψ:

$$E\Psi = \frac{p^2}{2m}\Psi + V\Psi$$

Next, we perform 'operator replacement' as follows.

$$E \rightarrow i\hbar\frac{\partial}{\partial t} \tag{4.1}$$

$$\vec{p} \rightarrow -i\hbar\nabla \tag{4.2}$$

In spacetime, these can both be written elegantly as

$$p_a \rightarrow i\hbar\nabla_a \tag{4.3}$$

The apparent sign difference is simply a matter of covector as opposed to vector, and is unimportant. Now, why have we made these definitions? The main reason is, they work. Schrodinger, who derived the equation in a different way (replacing the wave function in the classical action with the natural log of the wave function), chose this equation because it gave the right answers for hydrogen. Making the replacements, we arrive at

$$i\hbar\frac{partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi \tag{4.4}$$

This is the famous Schrodinger wave equation.

The equation can be motivated somewhat by looking at plane waves. For example, it's evident that

$$\Psi = Ae^{i(\vec{k}\cdot\vec{r}-\omega t)} \tag{4.5}$$

is a solution of the Schrodinger equation when $V = 0$. This complex wave appears in many areas of physics and mathematics, and is at the foundation of fourier analysis. Since the Schrodinger equation is linear, linear combinations of solutions of this kind can be summed, and through these series, functions with virtually any desired property can be constructed. Combined with the fact that V is essentially a user selectable function, and it perhaps seems less unlikely that the equation could fail to give physically useful results. Nonetheless, unphysical solutions crop up regularly, and are dismissed. This is most glaringly the case in the hydrogen atom.

The Schrodinger equation can be separated readily. Set

$$\Psi(\vec{r},t) = e^{-i\omega t}\psi(\vec{r}) \tag{4.6}$$

where $\omega = E/\hbar$. Plugging this into Schrodinger's equation, carrying out the partial derivatives, and canceling the factors of $e^{-i\omega t}$ on either side gives

$$E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \tag{4.7}$$

This is called the time-independent Schrodinger equation, and most of the time will represent the primary point of departure.

4.3 Simple Quantum Systems in One Dimension

4.4 Particle in a Box

Consider a particle trapped in a one dimensional box with infinitely high potential walls. The potential is given by

$$V = \begin{cases} \infty \text{of } x < 0//0 & 0 \le x \le L \\ 0 & x > L \end{cases}$$

In the region between the walls, Schrodinger's time-independent equation reads

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dt^2} = E\psi \tag{4.8}$$

Since V=0 in between the walls. Rearranging, this becomes the harmonic oscillator equation:

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

For $E > 0$, the solution therefore reads:

$$\psi = A\sin(kx) + B\cos(kx) \tag{4.9}$$

where A and B are constants. Applying boundary conditions

4.5 Quantum Systems in Two and Three Dimensions

4.6 Tunneling

4.7 The Hydrogen Atom

4.8 Quantum Scattering

4.9 Fermi's Golden Rule

4.10 Approximate Methods

4.11 Examples

Chapter 5

Relativistic Quantum Mechanics

One of the shortcomings of Schrodinger's Quantum Mechanics was the fact that it was not invariant under Lorentz transformations. This is considered a serious defect, since it seems to say that a moving observer might find some alteration in the laws of physics. And this doesn't make a lot of sense. For this reason, researchers sought relativistic theories.

5.0.1 The Klein-Gordon Equation

The Klein-Gordon equation was actually discovered by Schrodinger, even before he came up with his wave equation, but he rejected it because it didn't give the right spectrum for hydrogen. Starting with the energy-momentum equation,

$$E^2 - p^2c^2 = m^2c^4$$

and using the standard operator substitution,

$$E \to i\hbar\frac{\partial}{\partial t}$$
$$p \to -i\hbar\nabla$$

the Klein-Gordon equation is obtained:

$$\frac{\partial^2\Psi}{\partial t^2} - \nabla^2\Psi + \frac{m^2c^2}{\hbar^2}\Psi = 0 \tag{5.1}$$

To introduce a scalar potential, V, the method is to let $E \to E - V$, followed by operator replacement. Using the tensor equations is somewhat preferable and more systematic. Then there is only a single replacement:

$$p_a \to i\hbar\nabla_a \tag{5.2}$$

which is substituted into the equation

$$\eta^{ab}p_a p_b = m^2c^2 \tag{5.3}$$

5.0.2 The Dirac Equation

One of the positive qualities of the Schrodinger equation is that it is first-order in time. That means that specifying Ψ at a single moment determined the subsequent evolution. The derivative initial condition, $\partial\Psi/\partial t$, need not be considered. So Dirac set out to find an equation that was first order, and which also satisfied the Lorentz transformations.

Chapter 6

Atomic Physics

Atomic Physics is about the detailed electronic structure of the atom, taking into account a variety of subtle effects that result in splittings in atomic spectra, as well as explaining the regularities in the periodic table.

Chapter 7

Nuclear Physics

Nuclear physics is the study of the nucleus of atoms and their reactions and interactions. It turns out that quantum mechanics plays a key role in determining the structure of the nucleus, just as it had determined energy levels in the electronic structure of atoms.

7.1 Structure of Nuclei

The nuclei of atoms contain two kinds of particles, protons and neutrons. The protons have positive charge equal in magnitude to the charge on an electron, while the neutrons are neutral. Outside the nucleus, neutrons decay with a half-life of about 12 minutes, according to

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{7.1}$$

where $\bar{\nu}_e$ is an antielectron neutrino. Protons, on the other hand, are stable against decay, apparently forever, though experiments indicated a lifetime greater than about $10^{34} years$.

There are several important facts about nuclei of different sizes. 1. The number of protons in a nucleus is usually about the same as the number of neutrons.
2. The density of nuclei is approximately constant, regardless of the size of the nucleus.
3.

7.1.1 Liquid Drop Model

7.1.2 Shell Model

7.1.3 Collective Model

7.2 Radiation

7.3 Fission and Fusion

7.3.1 Fission

7.3.2 Fusion

7.4 Advanced Propulsion

7.4.1 Nuclear Thermal Rockets

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Chapter 8

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Chapter 9

Cosmology