Modern Physics Test 2 QUANTUM

November 14, 2001

This part of the test is to take home. Ground rules: (1) Consult only the two recommended texts and your class notes. (2) Verbal consultations with other class members is okay, but sharing of written test documents or other materials is not. (3) Test must be returned on Tuesday.

1. Solve Schrodinger's equation for a Coulomb potential in one dimension, finding all the energies and the first three eigenstates. The equation to be solved is

$$-\frac{\hbar^2}{2\mu}\frac{d^2\psi}{dx^2} - \frac{ke^2}{x}\psi = E\psi$$

Perform the following steps, in this order.

A. Find the asymptotic solution, ψ_∞. (Neglect the 1/x term for large x, and remember that $E < 0$.)

B. Substitute $\psi = f(x)\psi_\infty$, perform derivatives, cancel terms as needed.

C. Simplify the equation, by setting $y = ax$, and use the basis theorem to transform the derivative operators. Use this parameter a to eliminate junky factors in front of the potential term. a will have units of reciprocal meters, so that y will be dimensionless.

D. Redefine the energy with $\mathcal{E} = -E/\bar{E}$, where $\bar{E}$ is a suitable combination of junky constants associated with the energy on the right hand side. If you have done this correctly, then $\mathcal{E}$ will be dimensionless, and 'positive definite'.

E. Do a power series solution. Set

$$f = \sum_{n=0}^{\infty} A_n x^n$$

F. Obtain the recursion relationship for the A_n.

G. Eigenenergies are obtained by noting that this series must terminate, if the wave function is to have a finite integral over all x. Using the recursion formula, obtain these energies, and recast in electron volts.

H. Find the first three eigenfunctions corresponding to the ground state, the first and second excited states.

2. Find the probability of transmission through a Dirac Delta function potential, $V(x) = g\delta(x)$. To do this, represent a wave approaching from the left, together with the reflected portion, by

$$\psi_I = e^{ikx} + Re^{-ikx}$$

and the transmitted wave as

$$\psi_{II} = Te^{ikx}$$

Follow this process:

(A) Write down the boundary conditions for continuity of the wave function, and for the Dirac delta-function discontinuity of the first derivative.

(B) Use these two equations to solve for T and R (2 equations and 2 unknowns).

(C) Find the probabilities of transmission and reflection, $|T|^2$ and $|R|^2$.