Time in Quantum Mechanics and Temporal Coupling

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Time vs. time variable

Before discussing any issues related to the correlatedness of a quantum system's states in different moments of time, one should get a clear understanding of what quantum time is and what kinds of quantities are to be correlated. This is a non-trivial question, and there is a common delusion that the time variable in the time-dependant Schrödinger equation is enough to incorporate temporal variations in quantum physics. Most physicists do not ponder much on the fundamental ideas of quantum mechanics, and those who do usually get trapped in some primitive philosophy, introducing non-physical entities in physical theory (e.g. consciousness), and substituting investigation into the applicability of different theoretical models with void references to all sorts of imaginary demons. One cannot avoid questions about the physical time in quantum mechanics, if applications to inhomogeneous physical processes are to be considered.

To start with, let us observe that no single variable in quantum mechanics is intended to provide an observable property of the system, and it is operators that serve to represent the true observables, with their eigenvalues providing the complete list of possible cases. Mathematically, in the matrix representation, one can diagonalize any self-conjugate operator Q, thus selecting the description of the system in the Q-representation among all the other possible representation. In Q-representation, the eigenvalues q are used to *parametrize* the states of the system; however, they still have no relation to the selection of a particular mathematical representation of the eigenvectors, which still may be chosen from the infinity of the isomorphic mathematical spaces. Thus, for the representation diagonal in the total energy of a two-electron system, we can use either coordinate- or momentum- dependent wavefunction, or a mixed representation, with one electron described in the coordinate space, and another in the momentum space, as in Wigner approach. We can also get rid of any coordinate variables at all, using the secondary quantization techniques.

To summarize, the option of expressing the Schrödinger equation in spatial coordinate variables does not imply that there will be any spatial dependencies or correlations. Similarly, ***introducing the time variable in state vectors or operators will not necessarily lead to any observable temporal effects.*** In other words, it does not matter whether time-dependent or stationary formalism is employed — the result will be the same, as long as all we observe is atomic spectra. The is a matter of Fourier transformation, which is a simple linear operation that cannot introduce any new effects in the system's behavior. The system may be very complex and highly non-stationary — it does not change anything for a macroscopic observer, who will only detect characteristic shapes in the spectra.

This ascends to the fundamental paradigm of quantum mechanics, which I would refer to as the **Asymptotic Principle** (AP). The AP states that the observer is infinite compared to the microscopic system, so that all observer's influence on a quantum system must be contained in the asymptotic conditions at the spatial and temporal infinity, and the infinite space-time of the quantum system is all contained in a space-time point of the observer, so that it is only the asymptotic properties of the system that can be macroscopically detected. That is, for an observer, everything that happens in the quantum system occurs *instantly*, and *in the same spatial point*. Consequently, space and time variables used to describe the inside of the quantum system, its microscopic space-time, *cannot be physical observables*. Inversely, all cases of spatial or temporal description one could find in quantum experiment refer to *macroscopic* space and time, which can rarely be directly related to the variables in wave functions.

The macroscopic order one can observe in specially prepared coherent states is an only apparent contradiction to the AP. There is no significant modification of the basic experimental procedure, spectral measurement, and it is only the types of the spectra measured that may change. However, the very possibility of quite different spectral measurements may be useful to discover more about the process inside the quantum system that can never be directly observed. For instance, time spectra can be measured as well as energy spectra (extending the well known flight time techniques), which opens the opportunities for various industrial technologies based on the difference in the system's behavior in different times (controlled chemical reactions, quantum computing etc.)

For an observer, the processes inside a quantum system are always adiabatic. Mathematically this is expressed by that only the full scattering matrix $S = U(+\infty, -\infty)$ is associated with any observable effects, and never the partial evolution operators $U(t', t)$.

In principle, a quantum system does not need to be microscopic. Being a quantum system only means a specific mode of involvement in the interaction with other systems, when the characteristic time of external interactions is much greater than the typical times of internal relaxation, but comparable to the coherence time of the system.

Alternatively, energy exchange with the external world is much less than energy transfer inside the system, but comparable with the transition energies. A quite macroscopic system may well be described with quantum theory; for instance a human brain, an ecosystem, stock market, or a neutron star.

One might wonder how the AP could be valid for quantum description of plasma dynamics, phase transitions etc. According to the AP, all such time dependencies must be macroscopic, compared to the elementary processes in the medium, and therefore the collective effects observed in such systems have nothing to do with quantum physics proper. Well, collective effects can exist in macroscopic systems too, and almost any non-linear theory will readily provide numerous examples. However, there are physical models that explicitly include time-dependent terms in the equations of microscopic dynamics, to describe the influence of the medium on the elementary processes in it. Such models are in fact employing a variety of ***the hierarchical approach*** to the description of physical and other systems (e.g. the influence of the slow changing glial potentials on the spike activity of neurons in the brain). That is, in a hierarchical system, the equations of lower level dynamics can include a number of parameters representing higher level states, which are mathematically related to some integral characteristics of the lower level dynamics, averaging etc. In a real system, there may be many levels, so that each lower level will determine the characteristics available at the higher levels, and each higher level modifies the dynamics of the lower levels[1].

Hierarchical time

The hierarchical approach gives clues to constructing consistent models of temporal variations in quantum (and other) systems. We accept that physically meaningful time is associated with the sequences of events *of the same level*. Qualitatively, time manifests itself as sequencing. In the general case, this qualitative temporality yet does not allow time measurement. Under certain conditions, however, one can compare two sequenced processes *of the same level*, relating the events in one sequence to the events of the other; the latter can hence be considered as a sequence of *time labels* (clock ticks), and the same series of time labels can be used to measure time of quite different events, provided they belong to the same level of hierarchy, that is, the duration of event (the characteristic time of the virtual processes constituting the event) is much less than the period of clock ticks, and the lifetime of the event (the characteristic time of its influence on various objects of that level) is comparable with that period[2].

This makes it clear that the quality of time (the order of physical events) is absolute, while the quantity of time (the number of clock ticks between events) is relative and depends on the clock used. The former has to do with causality, the latter lies in the basis of the possible synthesis of quantum physics and relativity — the problem that is yet very far from being solved. As one can see now, the difficulties come from the incorrect transfer of the clock model from classical to quantum physics: Einstein fancied a quite definite type of timing mechanism, which simply cannot exist in quantum systems. As soon as we find the processes that could serve as quantum clocks, we immediately become able to build a consistently relativistic quantum theory.

Different levels of hierarchy will have different clocks. Physically, the levels are distinguished by the qualitative difference of their clocking intervals (that is, the typical rates of physical events): between two sequential ticks of the higher level clock, there may be many ticks of the lower level clock — the mathematical abstraction of this level difference is infinity: an infinite number of lower level events can fit into one higher level moment — they all proceed within one clock tick.

Introducing time in many-body systems has its peculiarities. In general, every body in the system has its internal motion, related to the corresponding time scales. The very possibility of considering several bodies as components of the same system implies that this internal motion does not much interfere with the interactions among the bodies, and the integrity of each body should not be violated by any external interactions. On the other hand, different bodies may differently interact with the other bodies in the system. If the characteristic times of such interactions are comparable with the clock period, they all belong to the same level, and there should be no problems in introducing time common to all the subsystems — the universal time. This is the most frequent approach in both classical and quantum physics, and it can significantly simplify calculations. If some of the bodies are interacting with the rest of the system much faster than the other bodies, a correct description would make the system hierarchical, describing the effect of the faster processes as averaged fluctuations (each fluctuation occurring in no time as regarded from the higher level). If there is a body that interacts with the rest of the system much slower than the other bodies, it should be accounted for as a higher level parameter. A

[1] There are different types of hierarchical systems. In weakly coupled systems, only the neighboring levels get entangled, while in strongly coupled systems there may be mutual influence through several levels.

[2] It should be stressed that the clock period is usually significantly shorter than the times measured, to allow for better measurement accuracy. However, it still must be comparable with typical times on that level, and a physical "time point" is different from a mathematical point, the latter only being a useful abstraction.

hierarchical system implies several scales of time, and one should be always aware of the current level of consideration, to avoid term confusion[3].

The distinction of the levels in a hierarchical system is not absolute. It depends on the hierarchy of the clocks used. The availability of a physical process that can be used for time labels determines the presence of the corresponding level in the hierarchy. Such timing events (clock ticks) may be external to the system, enabling absolute time measurements; however, in a many-body system, some interactions within the system could be used for timing, and this choice may be not unique.

Synchronization, correlation and coupling

Synchronization is literally translated as bringing to the same time. Unsynchronized events occur independently of each other, and they do not imply any common temporality at all. In the most general sense, synchronization means that two events become comparable through their relation to the same time scale. As soon as we can tell, which of the two events occurs earlier, or whether they occur simultaneously, the events are synchronized.

The weakest form of synchronization is mere existing in the same time scale. This is not so trivial as it may seem at the first glance. Thus, mathematically, the Cartesian product of the two independent time dimensions gets projected onto a one-dimensional manifold, different mappings describing different ways of establishing common temporality. In atomic physics, this would mean that the same time variable is used for all reacting particles, which is a very strong assumption valid only for reactions of a definite type.

Stronger synchronization may require event ordering. For instance, an atomic state cannot decay until it is formed, and an electron cannot be captured until it is emitted. This is an example of internal synchronization. External synchronization implies the existence of another system that triggers certain events depending on the original system's output. For instance, elementary processes in dense plasma result in microfield changes that may trigger specific atomic reactions. As it can be seen, external synchronization is only possible in hierarchical systems.

Synchronization is of crucial importance for industry. Biotechnology is impossible without carefully ordered sequences of quantum events (chemical reactions), and quantum computing requires strict synchronization of all the operations.

Correlation is the opposite of synchronization in that it compares two distinct systems without demanding their synchronization. Being correlated only means that occurrence of certain events in one system implies occurrence of certain events in another system, regardless of their order. In particular, one can speak about correlation within a physical system, between its different parts. It may seem that, thus defined, correlation describes only the spatial properties of the system. This not so, as the example of auto-correlation indicates: the events in the same system in different time points become related to each other. However, in auto-correlation, the system at two different time points is taken in simultaneity, and the order of the time points is of no importance. Mathematically this is expressed in that auto-correlation functions are even:

$$c(\tau) = \int f(t) f(t+\tau) dt = \int f(t-\tau) f(t) dt = c(-\tau)$$

There may also be synchronization without correlation, like in the case of cascades in atomic reactions. For instance, if an excited state has radiationally decayed on a metastable level, this level may further be depopulated by a charged projectile, the two processes being practically independent of each other.

Coupling syntheses two physical systems in one, demanding both synchronization and correlation. This is correlation through synchronization, and synchronization with correlation. Coupling depends on the kinematic conditions and demands a definite sequence of interactions. In atomic systems, the dominant parts of the interaction potential determine the realizable types of coupling. For instance, in a three-body system, there are different ways of binding the bodies in a whole: A+B+C, (A+B)+C, A+(B+C), (A+B+C); the preferable way depends on relative interaction strengths and may change in the course of atomic reaction due to kinematic of dynamic effects (re-coupling).

In a hierarchical system, there may be different levels of coupling. For example, in a two-electron atomic system, the electrons may be *weakly coupled* through the higher level, via the average field induced by each electron's motion (self-consistent equations); alternatively, there may be *strong coupling* through direct Coulomb interaction of atomic electrons.

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[3] Physicists are accustomed to spatially distinguishing the individual bodies in a many-body system, which ascends to the traditional paradigm of a massive point in classical mechanics. However, similarly to the hierarchy of time scales, one could consider the hierarchy of spatial scales, and indicate the cases when the traditional Cartesian product of individual configuration spaces will not adequately describe the geometry of the system. The Faddeyev method might serve as an example from atomic physics.

Temporal coupling assumes that the states of an atomic system in different time points (considered as two different systems) become correlated and synchronized. This normally means that there are two processes in the system that have comparable characteristic times, and the phase of on process would influence the other. There may also be weak coupling (*interference*), when it is through a common higher level (e.g. the observer) that the two processes communicate, or strong coupling (*interlace*).[4]

One space, one time?

Before considering the quantum time levels, the dimensionality problem should be touched in brief. In non-relativistic classical mechanics, a system of N bodies is characterized with $3N$ spatial coordinates, $3N$ momenta, and only one time coordinate. This model implies that there is a universal 3-dimensional space, and the positions of the bodies in this space are observable at any time. That is, both space and time are *associated with the observer*, rather than physical bodies themselves. Changing the observer results in another reference frame, retaining the same dimensions, so that different observers can compare their observations, using the standard transition formulas. In principle, the observer can be attached to one of the bodies in the system, but this is not necessary, since the observer is anyway external to the system itself.

Relativistic mechanics does not add anything new, except that Galilean transition rules become replaced with Lorentz transformation, involving time along with the spatial coordinates. All the reference frames are still independent of the motion of any physical bodies, depending only on the motion of the observer.[5] In other words, it is implicitly assumed that the observer can construct the reference frame much quicker than the state of the system will considerable change. Since the possible speeds are supposed to be limited, this means that the observer is negligibly small, compared to the system studied. This hints to the presence of a hierarchical structure, which has yet never been considered in any detail.

In the same way, quantum physics assumes that there is one 3+1-dimensional space time associated with the observer, and all the physical bodies (particles) are believed to move in this common space-time. However, unlike in the relativistic approach, the observer is supposed to be much larger than the system studied, so that any changes in the system happen much quicker than the observer can move.[6] This means that the observer cannot detect any inner motion of the system and can only conclude about it by its asymptotic manifestations, as demanded by the AP. Alternatively, one can achieve quantum behavior in macroscopic systems, if the speeds of the processes in the system are infinite, compared to the speed of observation. The difference of the relativistic and quantum paradigms is the main source of difficulties in relativistic quantum theory.

The common feature of classical, special relativistic and quantum theory is that space-time does not depend on the motion of the physical bodies (particles), being common to them all.[7] In other words, in modern physics, many-body theories are only concerned in the projection of the motion onto the observer's space-time, rather than on the motion of bodies themselves. To make one more step away from the anthropocentric description, one could consider more levels of hierarchy, with the bodies of a higher level playing the role of the observer for the adjacent lower level. Alternatively, one could picture a few different observers, each representing a different view of the same system; these individual observations become integrated in a higher-level observer, who can also observe the mutual influence of lower-level observations, and their transformations. In fact, sun an approach is implicitly introduced in quantum mechanics, to describe various re-coupling phenomena. For instance, considering different asymptotics for incoming and outgoing states is already a deviation from the single-

[4] This distinction requires some explanation. The usual notion of interfering channels in atomic reactions is associated with summing up the amplitudes corresponding to different reaction mechanisms, and it is only in the squared absolute value that the characteristic cross-terms appear. This squaring procedure is the usual way to produce the observable quantities from the amplitudes, and hence the processes that simply sum on the microscopic level become interfering in macroscopic observation. With interlace, already on the quantum level there is no linearity, and no amplitude summation. This is analogous to the well-known procedure of continuum diagonalization in Fano-Pratts theory: the normalization factor introduces non-linearity, which may be important to obtain correct autoionization widths.

[5] This motion is relative to the motion of other observers, but not to the motion of the physical system to describe.

[6] One might point at an apparent contradiction: some metastable states may exist for quite a while, on the macroscopic scale. However, the very process of decay is still much faster than its detection, and the waiting time does not count. Interaction strength only determines the frequencies (rates) of quantum events, but not their rapidity.

[7] Even general relativity is not enough to dissolve the distinction between the observer and the system observed, only introducing an idea of a heavily structured reference frame, which, however, is still locally 3+1-dimensioned and independent of the moving bodies.

observer approach, and there are conceptual problems due to the inconsistency of the "plain" mathematical model, trying to use the same Hamiltonian for both the initial and final states of reaction.

To summarize, correct description of structural changes in an atomic system requires a multilevel approach, the higher levels integrating the different views of the lower-level dynamics. One could regard it as a combination of the conventional state vector and density matrix formalisms: different representations of the system and their correlations are modeled with a kind of density matrix, while every particular representation assumes a state vector.[8]

Atomic time

In quantum theory, space and time of a quantum system are not directly related to the space-time of the observer; according to the AP, all the quantum events occur in their own space-time, which is infinite for the quantum system, but infinitesimal for the observer. This is a typical hierarchical system, and all the above considerations about hierarchical time should be applicable.

However, even in the quantum space-time, there may be different levels, which will be described with their characteristic times. It is only in the case of very strong coupling that internal hierarchical structures cannot develop. As soon as there is difference in interaction strength, perturbation theory can be applied to the weaker interaction, implying a hierarchy of virtual interactions between the initial and final states. Such hierarchical structures cannot be directly observed, only manifesting themselves in the fine details of the observable asymptotic behavior of the system.

This is an important feature of any hierarchy: the lower levels are always represented on the higher level, and hence can be studied on the basis of the higher-level observables.

However, the existence of different space-time scales does not yet imply any difference in spatial or temporal structure of the individual particles. Rather, space-time is a characteristic of a definite level, its integral property, and there are no separate particles communicating about time, at least in the currently known models. All the particle move in the same space-time, and any synchronization, correlation or coupling can only be established through a higher level. Conceptual difficulties associated with the idea of several interacting particles, and reference frames associated with them, have not yet been resolved, and that is why the dynamic approach, based on equations of motion, is of a restricted applicability in atomic theory, the formal scattering matrix formalism absolutely dominating in the relativistic domain.

In non-relativistic quantum mechanics, one could directly introduce multiple time variables in the Schrödinger equation:

$$\left(\sum_k H_k(x_k,t_k)+V(x_1,t_1;x_2,t_2;...)\right)\psi(x_1,t_1;x_2,t_2;...)=i\sum_k\frac{\partial}{\partial t_k}\psi(x_1,t_1;x_2,t_2;...)\,.$$

In the limit case of time-independent V and

$$\psi(x_1,t_1;x_2,t_2;...)=\prod e^{iE_kt_k}\psi_k(x_k)\,,$$

this equation will obviously reduce to the ordinary the Schrödinger equation, and it could open interesting theoretical perspectives, especially in application to atoms in variable external fields, and possibly quantum event sequencing in solids, or bio-molecules.

Within the traditional one-time formalism, most temporal coupling effects arise from the perturbation theory expansions. As indicated, the perturbation theory approach is nothing but an attempt to introduce a hierarchical structure in a non-hierarchical system; that is why the applicability of perturbation techniques may be sometimes questionable. The basic idea behind perturbation theory is that all the interactions within the system, or interactions with external fields and particles, can be ordered in time, though the exact times of individual acts of interaction cannot be determined, since they all occur virtually, within a single time point of the observer. The observer will only see an asymptotic projection of the inner sequences of interactions averaged over all possible virtual interaction time values. Formally, in the standard scattering theory, one would introduce the evolution operator and wavefunction (state vector) in the interaction representation:

$$\Psi(t)=U(t,t_0)\Psi(t_0)\,,$$

with $\Psi(-\infty)=\Psi_{in}$ and $\Psi(+\infty)=\Psi_{out}$ being the initial and final asymptotic states, and $S=U(\infty,-\infty)$ being the scattering matrix, which is related to the observable quantities. The perturbation expansion for the evolution

[8] Yet another level is needed to model the inner motion of the particles; this will lead us to spinor components.

operator is commonly written as

$$U(t,t_0)=\sum_{n=0}^{\infty}U_n(t,t_0)\,,$$

with

$$U_n(t,t_0)\sim\int_{t_0}^{t}V(t_1)dt_1\int_{t_0}^{t_1}V(t_2)dt_2...\int_{t_0}^{t_{n-1}}V(t_n)dt_n=\frac{1}{n!}\int_{t_0}^{t}dt_1\int_{t_0}^{t}dt_2...\int_{t_0}^{t}dt_nT\{V(t_1)V(t_2)...V(t_n)\}.$$

That is, we distinguish reactions with different number of interactions between the initial and final states as belonging to different levels of hierarchy; this is usually justified by that interaction V is small enough, and the sequences of more interactions are less likely to occur. Formally, one could introduce some average interaction strength $\overline{V}$, and consider it as small parameter, so that

$$V=\overline{V}\cdot\left(V/\overline{V}\right),\text{ and }U_n\sim\overline{V}^n/n!\,.$$

In reality, the picture may be much more complex, since there different atomic interaction of different strengths, which is often accounted for in an additive way:

$$V(t)=V_1(t)+V_2(t)+...$$

Strictly speaking, such an additivity is only possible for commuting interactions V_k; however, even in this simplest case the perturbation hierarchy becomes rather complicated, containing terms with the products of different powers of different interaction strength parameters.

It should be stressed, that quantum mechanics does not delve into details of every elementary act of interaction (quantum event): all such acts occur in no time, and it is only their sequences that can be considered. However, various renormalization techniques exist to partially sum the perturbation series over selected component interactions; this is a standard tool to fold the perturbation hierarchy. The resulting (normalized) interactions may be non-local, which is an indication of how lower-level structures can be introduced in a higher-level model. Indeed, the very non-locality of quantum mechanics in general is an instance of that hierarchical modeling.

One could observe that, for commuting $V(t)$, the ordering of quantum events formally disappears from the evolution operator:

$$U_n(t,t_0)\sim\int_{t_0}^{t}dt_1\int_{t_0}^{t}dt_2...\int_{t_0}^{t}dt_nT\{V(t_1)V(t_2)...V(t_n)\}\to\int_{t_0}^{t}dt_1V(t_1)\int_{t_0}^{t}dt_2V(t_2)...\int_{t_0}^{t}dt_nV(t_n)\,.$$

Quantum events occurring in different moments of time could be hence considered entirely uncorrelated. However, in general, this does not mean the absence of synchronization. Indeed, in quantum mechanics, operators do not have any physical meaning on themselves, and it is their matrix elements that describe various physical properties. In the perturbation series, such a matrix element will represent a particular transition from the initial to final state, through a definite sequence of intermediate states.[9] That is, in the uncorrelated limit, a quantum transition is described with the amplitude

$$\int_{t_0}^{t}dt_1\langle f|V(t_1)|s_1\rangle\int_{t_0}^{t}dt_2\langle s_1|V(t_2)|s_2\rangle...\int_{t_0}^{t}dt_n\langle s_{n-1}|V(t_n)|i\rangle\,,$$

which, obviously, can only include sequences of virtual transitions $\langle a|V|b\rangle$ that can be at least orderable in a chain, albeit there is no temporal correlation between such intermediate events. Moreover, the intermediate wavefunctions may depend on time themselves, which will add more synchronization in the system's dynamics.

As long as quantum events are structureless, the system is time-invariant in every single act of interaction, and each intermediate state can be characterized with a definite energy. This allows to rewrite the whole theory in the stationary formalism, without loosing any generality, merely applying the Fourier transform. For quantum systems with local interactions, stationary description is equivalent to the time-dependent formalism, and all the results obtained in one of them could be as well derived in another.

The stationary formalism is convenient for applications, and it allows to introduce the important distinction between energy-conserving (*on-shell*) transitions and those without total energy conservation (*off-shell*

[9] In fact, an operator is nothing but an abstraction of transition, and that is why any statements about operators do not make sense without specifying the appropriate domains.

transitions). Thus, if the observer's time is uniform enough, the observable events, according to the AP, must be time-invariant, and hence the total energy must be conserved. Inversely, any on-shell transition can, in principle, be observed, and hence belongs to the topmost level in the hierarchy of quantum events. On the contrary, off-shell transitions can only be virtual; this could be interpreted as if the off-shell events belonged to a lower level of hierarchy, and their time should be measured in a finer scale. Following the above idea about multilevel observer, one would conclude that the on-shell states form the level of "inner observer", in addition to the usual asymptotic macroscopic observer.

That is, in atomic physics, there is an implicitly assumed *hierarchy of time* for every particular atomic reaction, so that every level is characterized with a specific time scale, being virtual for the upper levels. One could summarize it as follows.

1. Virtual sequencing — Perturbation time

This is the deepest level attainable in a quantum theory. It refers to ordering the off-shell transitions within an on-shell process. However, the characteristic times of that levels are still much greater than the event time, and every act of interaction is still a point in perturbation time. Most probably, the influence of this level on the observable characteristics will be mainly quantitative, without drastically changing the shapes of the spectra.

In terms of statistics, all processes of this levels could be treated as fluctuations. They are to be carefully avoided in any applications demanding microscopic synchronization (such as quantum computing). For instance, as calculations show, autoionization width of the $3s3p$ resonance in helium is sensitive to the off-shell contribution; this implies that the processes involving this state cannot be used in quantum computing and similar applications.

2. Physical sequencing — Quenching time

On this level, the virtual sequences of quantum transitions are formed, distinguishing a number of interfering (coherently summed) reaction channels. The processes of this level are quasi-stationary, energy conserving (on-shell). The collection of interfering reaction channels determines the qualitative behavior of observable spectra. Under certain conditions, such processes can be made observable, or controlled, using external agents.

On this level, the system's behavior is well synchronized in each reaction channel, but it is problematic whether the dominance of a particular channel can be maintained. The cases of critical behavior (mutual enhancement or cancellation of different channels, qualitative changes in angular distributions of reaction products, their polirization etc.) are the most important for applications.

While the number of interfering reaction channels may influence the shapes of spectral resonances, not all channels are equally important in resonance formation. As a rule, a few principal channels dominate, the rest merely adding minor corrections. This circumstance underlies the efficient usage of the various forms of channel coupling models (to be discussed below).

The general direction of developing experimental techniques to achieve ultra-high resolution is in adjusting the conditions of reaction so that different channels would dominate in the total transition amplitude, resulting in experimentally detectable changes in resonance profiles; interpreting such changes within a definite theoretical model allows to obtain information about each interfering process individually [Godunov et al., *J. Phys. B* **33**, (2000)].

3. Microevolution — Reaction time

When there are no alternative reaction channels, reaction takes a quite definite path, resulting in a *cascade* of transitions, every one of which can be independently detected by the observer using different techniques. In particular, there may be only one detectable transition, with no other channels to interfere.

A sequence of such quantum events still occurs in no time for the observer, but now each transition implies completed previous transitions, and their details are irrelevant for that next transition. This is equivalent to the presence of a number of micro-observers, detecting individual transitions as soon as they take place, but reporting to the macroscopic observer all at once.

Cascades has long since become one of the main instruments in studying atomic structures. Implementation of quantum computers on the level of microevolution seems rather promising too.

It should be stressed that there is no absolute distinction between cascades and physical sequencing. Thus, the interference of reaction channels may vanish under certain observation conditions (e.g., in angular distributions and angular correlations), and the system may well be described with reaction time in these condition, otherwise remaining coupled.

4. Macroevolution — Observable time

So far, only spectral measurements have been considered. However, in some cases, different sequences of quantum events can be resolved by the macroscopic observer, with exact timing. For instance, certain particles that can only be emitted in specific transitions (microevolution), and these particles are be detected sequentially, or in a number of correlated observations in different spatial points. This implies a chain of quantum events occurring in the macroscopic time.

The presence of macroscopic evolution in microscopic events may be one of the most powerful means of quantum control. Such a presence may be introduced via applying variable or spatially inhomogeneous external fields. The feasibility of control over the spatial or temporal variations of such fields is an essential requirement. For instance, a proton scattered on an atomic target can produce rapid field variations in the interaction region, but these variations cannot be controlled by the observer, occurring in the perturbation or quenching time; as a result, no macroscopic time variations can be detected.

Channel Coupling Approximation

According to the AP, it is the asymptotic parameters of quantum motion that can only be observed. That is, the time-dependent wave function in the non-stationary Schrödinger equation

$$H(t)\Psi(t) = i\frac{\partial\Psi(t)}{\partial t}$$

does not describe any physical evolution in the macroscopic time, and it is only its limit values

$$\Psi(-\infty) = \Psi_{\text{in}} \text{ and } \Psi(+\infty) = \Psi_{\text{out}}$$

that may be related to measurable quantities. Since only a few reaction outcomes can practically be measured, it seems natural to approximate the exact dynamics of the system with the evolution of the projection of the full virtual wave function $\Psi(t)$ onto the spaces spanned by the asymptotic states Ψ_{in} and Ψ_{out}:

$$\Psi(t) = \sum_{\tau} a_{\tau}^{\text{in/out}} \Psi_{\tau,\text{in/out}} \ .$$

Substituting this expansion in the Schrödinger equation, and calculating its matrix elements between the asymptotic states, one obtains a set of equations for *populations* $a_{\tau}^{\text{in/out}}$, rather than the virtual quantum state $\Psi(t)$. This method is based on the paradigm of interfering reaction channels, and hence can be called the *method of dynamic channel coupling*.

In the stationary description, one can similarly introduce the projections

$$\Psi_{\tau}(E) = \sum_{\xi} \int dE' \, \Psi_{\xi,\text{in/out}}(E') C_{\xi\tau}^{\text{in/out}}(E', E)$$

of the full energy dependent state $\Psi_{\tau}(E)$ onto the space spanned by the asymptotic states $\Psi_{\xi,\text{in/out}}(E')$, and substitute it into the stationary Schrödinger equation

$$H\Psi_{\tau}(E) = E\Psi_{\tau}(E) \ ,$$

to obtain equations for the expansion coefficients. This approach is known as the *method of close coupling* (in the integral formulation). As indicated, the two versions of the method of channel coupling (time-dependent and stationary) are equivalent as long as interactions remain local.

One could notice the resemblance of the channel coupling approximation (CCA) to the secondary quantization approach. The only difference is in the level of consideration: while CCA is applicable starting from the level of virtual sequencing, secondary quantization models instantaneous state formation in a quantum event, thus referring to a yet lower levels of hierarchy and being a kind of a *folded* CCA.

Obviously, the time-dependent and stationary approaches can be combined, to account for some interactions in quantum state populations, and some other interactions in the structure of the states themselves. Such a combined method would be preferable for hierarchical systems, when observer-controlled time dependencies are involved along with virtual transitions.

As an interesting generalization, consider asymptotic states dependent on time:

$$\Psi_{\text{in}} = \Psi_{\text{in}}(t) \text{ and } \Psi_{\text{out}} = \Psi_{\text{out}}(t) \ .$$

This parameter corresponds to observer time, which has yet to be related to any microscopic temporality. The

generalized Schrödinger equation then reads as

$$H(t'|t)\Psi(t'|t) = i\frac{\partial\Psi(t'|t)}{\partial t'},$$

and now it is directly applicable to the description of macroevolution. Any number of co-existing time levels could be modeled than way. In practical calculations, the dependence on the macroscopic time t can be replaced with some equivalent parametric dependence (projectile velocity, impact parameter etc.). If no relation is established between macroscopic and microscopic time variables, this model describes *adiabatic* macroevolution of a quantum system, when all the transitory processes are completed at any moment of observer time; this corresponds to $t', t'' \to \pm\infty$ in the microevolution operator $U(t', t'')$.

Temporal Coupling in Atoms?

Since, in the standard non-relativistic theory, there is a single time for all the particles comprising the quantum system, one cannot directly consider electrons communicating about time. It may seem that no temporal coupling is possible at all in such a theory. This suspicion is well justified for unstructured processes, which can be reduced to the first-order perturbation theory; however, sequencing and temporal correlation are inherent to any model involving multiple reaction channels.

In a local theory, one cannot directly compare atomic states $|t_1\rangle$ and $|t_2\rangle$ related to different time points. The natural way to bring them to a common basis is using the microevolution operator:

$$\Psi(t) = U(t, t_0)\Psi(t_0).$$

The matrix elements $\langle t_1|U(t_1, t_2)|t_2\rangle$ could used to characterize temporal coupling. That is, two temporal states of an atomic system are uncoupled if there is no physical process that could transform one of them to another. This approach seems physically sensible, and compatible with causality. Using the identity:

$$U(t_1, t_2) = U(t_1, \infty)U(\infty, t_2),$$

we also obtain

$$\langle t_1|U(t_1, t_2)|t_2\rangle = \langle t_1|U(t_1, \infty)U(\infty, t_2)|t_2\rangle = \sum_{\alpha}\langle t_1|U(t_1, \infty)|\alpha\rangle\langle\alpha|U(\infty, t_2)|t_2\rangle,$$

where the index α enumerates the asymptotic states of the system. This leads to an important corollary that *atomic states with common asymptotic are temporally coupled*. However, there may be different levels of coupling, and it is the differences between coupling types that are of interest for applications.

Expanding the evolution operator in the perturbation series

$$U(t_1, t_2) = 1 + U_1(t_1, t_2) + U_2(t_1, t_2) + \ldots,$$

we obtain

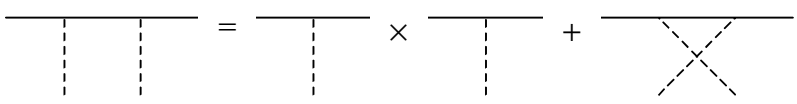
$$\langle t_1|U(t_1, t_2)|t_2\rangle = \langle t_1|t_2\rangle + \langle t_1|U_1(t_1, t_2)|t_2\rangle + \langle t_1|U_2(t_1, t_2)|t_2\rangle + \ldots$$

The zero-order term in this expansion describes the trivial case of self-correlation. The first order term is related to direct transitions, and it does not imply any sequencing. Temporal correlation in the first order is revertible and asynchronous. Starting from the second order, various sequencing effects can be involved, and atomic transitions may be synchronized.

The second order evolution operator could be decomposed into the reducible and irreducible parts:

$$U_2(t_1, t_2) = U_1(t_1, t_2) \times U_1(t_1, t_2) + U_{2i}(t_1, t_2),$$

which might be diagrammatically expressed as



The first term in such a decomposition describes independent occurrence of two transitions, while the $U_{2i}(t_1, t_2)$ term accounts for strong temporal coupling (interlace), and it is mainly responsible for virtual sequencing.

The direct product of two first order evolution operators in the first term does not imply any synchronization of the first-order transitions; for sequencing, one has to project it onto a common level, thus introducing explicit

non-linearity. The possibility of sequencing is related to the presence of a higher level[10] integrating physically different entities. The product $U_1(t_1,t_2)\times U_1(t_1,t_2)$ cannot, in general, be reduced to $U_1(t_1,t_2)U_1(t_1,t_2)$, since the latter operator is expected to act on the states defined in different time points. Indeed, one can formally introduce an abstract complete set of quantum states (usually, the asymptotic states of the system) and use the expansion

$$\langle t_1|U_2(t_1,t_2)|t_2\rangle = \langle t_1|U_1(t_1,t_2)U_1(t_1,t_2)|t_2\rangle = \sum_\alpha \langle t_1|U_1(t_1,t_2)|\alpha\rangle\langle\alpha|U_1(t_1,t_2)|t_2\rangle$$

However, in the $\langle\alpha|U_1(t_1,t_2)|t_2\rangle$ amplitude, the states α are supposed to be taken at the moment t_1, while in the amplitude $\langle t_1|U_1(t_1,t_2)|\alpha\rangle$ they rather belong to t_2. The contradiction becomes hidden, when $t_{1,2}\to\pm\infty$, and one can use the same set of asymptotic states everywhere. This implies a higher-level observer, and that is how hierarchies get folded in nonlinear theories.

In the folded (non-linear) form, the $U_1(t_1,t_2)U_1(t_1,t_2)$ term in the decomposition of the second-order evolution operator allows for physical sequencing, or microevolution. In the hierarchical system considered, system states at any time can be projected onto asymptotic states, and we obtain

$$\langle t_1|U_2(t_1,t_2)|t_2\rangle = \sum_{\alpha\sigma\tau}\langle t_1|\sigma\rangle\langle\sigma|U_1(t_1,t_2)|\alpha\rangle\langle\alpha|U_1(t_1,t_2)|\tau\rangle\langle\tau|t_2\rangle$$

This expression may describe very complex interference; to simplify it, one could use the *independent time approximation* (ITA), assuming that

$$\langle\sigma|U_1(t_1,t_2)|\alpha\rangle\langle\alpha|U_1(t_1,t_2)|\tau\rangle \sim \delta_{\sigma\tau}\langle\sigma|U_1(t_1,t_2)|\alpha\rangle\langle\alpha|U_1(t_1,t_2)|\sigma\rangle,$$

or, alternatively, that

$$\langle\tau|t_2\rangle\langle t_1|\sigma\rangle \sim \delta_{\sigma\tau}\langle\sigma|t_2\rangle\langle t_1|\sigma\rangle.$$

Physically, this assumption means that the coupling of different transitions via a higher level occurs in no time (on the reaction time scale), and within the same quantum state[11]. In the ITA, we have

$$\langle t_1|U_2(t_1,t_2)|t_2\rangle = \sum_{\alpha\sigma}\langle t_1|\sigma\rangle\langle\sigma|U_1(t_1,t_2)|\alpha\rangle\langle\alpha|U_1(t_1,t_2)|\sigma\rangle\langle\sigma|t_2\rangle,$$

which becomes

$$\langle\sigma|U_2(t_1,t_2)|\sigma\rangle = \sum_\alpha\langle\sigma|U_1(t_1,t_2)|\alpha\rangle\langle\alpha|U_1(t_1,t_2)|\sigma\rangle$$

at $t_{1,2}\to\pm\infty$.

[10] Recall that, in mathematics, the space of possible paths on a manifold is different from that manifold, and the space of functions on some domain is of a higher cardinality than that domain. Only locally, in the linear case, there may be one-to-one correspondence between operators on some space and the points of that space.

[11] That is, the quantum state has no time to change, and the act of coupling does not interfere with atomic reaction itself. In other words, ITA assumes independence of time scales on different levels of hierarchy, rather than independence of quantum events.